# VERY WEAK ZERO ONE LAW FOR RANDOM GRAPHS WITH ORDER AND RANDOM BINARY FUNCTIONS SH548

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## § 1. Introduction

Let $G_<(n,p)$ denote the usual random graph $G(n,p)$ on a totally ordered set of $n$ vertices. (We naturally think of the vertex set as $1,\dots,n$ with the usual $<$). We will fix $p=\frac{1}{2}$ for definiteness. Let $L^<$ denote the first order language with predicates equality $(x=y)$, adjacency $(x\sim y)$ and less than $(x<y)$. For any sentence $A$ in $L^<$ let $f(n)=f_A(n)$ denote the probability that the random $G_<(n,p)$ has property $A$. It is known Compton, Henson and Shelah [CHSh:245] that there are $A$ for which $f(n)$ does not converge.

Here we show what is called a *very weak zero-one law* (from [Sh:463]):

**Theorem 1.1.** *For every $A$ in language $L^<$* {0.1}

$$\lim_{n\to\infty}(f_A(n+1)-f_A(n))=0.$$

Note, as an extreme example, that this implies the nonexistence of a sentence $A$ holding with probability $1-o(1)$ when $n$ is even and with probability $o(1)$ when $n$ is odd (as in Kaufman, Shelah [KfSh:201]).

In §2 we give the proof, based on a circuit complexity result.

In §3 we prove that result, which is very close to the now classic theorem that parity cannot be given by an $AC^0$ circuit.

In §4 we give a very weak zero-one law for random two-place functions. The proof is very similar, the random function theorem being perhaps of more interest to logicians, the random graph theorem to discrete mathematicians.

The reader should thank Joel Spencer who totally rewrote the paper (using the computer science jargon rather than the logicians one), and with some revisions up to the restatement in the proof of **??** but with **??**, this is the version presented here. We thank the referee for comments on the exposition, and we thank Tomasz Łuczak and Joel Spencer for reminding me this problem on $G_<(n,p)$ in summer of 1993.

On a work continuing this of Boppana and Spencer see **??**(5).

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## § 2. The Proof

Let $G$ be a fixed graph on the ordered set $1,\ldots,2n+1$. For a property $A$ and for $i=n,n+1$ let $g(i)=g_{G,A}(i)$ denote the probability that $G\restriction_S$ satisfies $A$ where $S$ is chosen uniformly from all subsets of $1,\ldots,2n+1$ of size precisely $i$. We shall show

{0.2} **Theorem 2.1.** $g(n+1)-g(n)=o(1)$.

More precisely, given $A$ and $\epsilon>0$ there exists $n_0$ so that for any $G$ as above with $n\geq n_0$ we have $|g(n+1)-g(n)|<\epsilon$.

We first show that Theorem 1.1 follows from Theorem 2.1. The idea is that a random $G_<(i,p)$ on $i=n$ or $n+1$ vertices is created by first taking a random $G_<(2n+1,p)$ and then restricting to a random set $S$ of size $i$. Thus (fixing $A$) $f(n),f(n+1)$ are the averages of $g_G(n),g_G(n+1)$ over all $G$. By Theorem 2.1 we have $g_{G,A}(n)-g_{G,A}(n+1)=o(1)$ for all $G$ and therefore their averages are only $o(1)$ apart.

Now we show Theorem 2.1. Fix $G,A$ as above. Let $P(S)$ be the Boolean value of the statement that $G|_S$ satisfies $A$. For $1\leq x\leq 2n+1$ let $z_x$ denote the Boolean value of "$x\in S$" so that $P(S)$ is a Boolean function of $z_1,\ldots,z_{2n+1}$. We claim this function has a particularly simple form. Any $A$ can be built up from primitives $x=y,x<y,x\sim y$ by $\wedge,\neg$ and, critically, $\exists_x$. As $G$ is fixed the primitives have values true or false. Let $\wedge,\neg$ be themselves. Consider $\exists_x W(x)$ where for each $1\leq x\leq 2n+1$ we let $W(x)$ on $G|_S$ is given by $W^*(x)$. Then $\exists_x W(x)$ has the interpretation $\exists_{x\in S}W(x)$ which is expressed as $\vee_{x=1}^{2n+1}(z_x\wedge W^*(x))$. For convenience we can be redundant and replace $\forall_x W(x)$ by $\bigwedge\limits_{x=1}^{2n+1}(z_x\Rightarrow W^*(x))$. For example $\forall_x\exists_y x\sim y$ becomes

$$\bigwedge_x[Z_x\to V_y(x\sim y\wedge Z_y)].$$

Thus $P(S)$ can be built up from $z_1,\ldots,z_{2n+1}$ be means of the standard $\neg,\wedge,\vee$ and $\wedge,\vee$ over (at most) $2n+1$ inputs. That is (see §3) $P(S)$ can be expressed by an $AC^0$ circuit over $z_1,\ldots,z_{2n+1}$ (of course with the number of levels bounded by the length $d_A$ of the sequence $A$ (can get less) and the number of nodes bounded by $d_An^{d_A}$). Now $g(i)$, for $i=n,n+1$, is the probability $P$ holds when a randomly chosen set of precisely $i$ of the $z$'s are set to True. From Theorem 3.1 below $g(n+1)-g(n)=o(1)$ giving Theorem **??** and hence Theorem 1.1.

## § 3. $AC^0$ Functions

We consider Boolean functions of $z_1, \dots, z_m$. (In our application $m = 2n+1$.) The functions $z_i, \neg z_i$, called literals, are the level 0 functions. A level $i+1$ function is the $\wedge$ or $\vee$ of polynomially many level $i$ functions. An $AC^0$ function is a level $d$ function for any constant $d$. By standard technical means we can express any $AC^0$ function in a "leveled" form so that the level $i+1$ functions used are either all $\wedge$s of level $i$ functions or all $\vee$s of level $i$ functions and the choice alternates with $i$ (at most doubling the number of levels). It is a classic result of circuit complexity that parity is not an $AC^0$ function. Let $C$ be an $AC^0$ function. For $0 \le i \le m$ let $f(i) = f_C(i)$ denote the probability $C$ holds when precisely $i$ of the $z_j$ are set to True and these $i$ are chosen randomly.

{0.3}

**Theorem 3.1.** $f(n+1) - f(n) = o(1)$.

A restriction $\rho$ is a partial function from the set $\{1, 2, \dots, 2n+1\}$ to the set $\{0, 1\}$. If for some $i \in \{1, 2, \dots, 2n+1\}$, the restriction $\rho$ is not defined we sometimes write $\rho(i) = *$.

Call a restriction $\rho$ balanced if $|\{i : \rho(i) = 0\}| = |\{i : \rho(i) = 1\}|$.

Now more fully the theorem says

- $(*)$ for every $\varepsilon$, $d$, $t$ there is $n_{\varepsilon,d,t}$ satisfying: if $n \ge n_{\varepsilon,d,t}$ and $C$ is an $AC^0$ Boolean circuit of $z_1, \dots, z_{2n+1}$ of level $\le d$ with $\le n^t$ nodes then $|f_C(n+1) - f_C(n)| < \varepsilon$.

This statement is proved by induction on $d$.

We choose the following

- $(i)$ $c_0 = (\ell n 2), c_0' = -(|n\frac{2\varepsilon}{3}/|n2) > 0$
- $(ii)$ $\varepsilon_\ell = \frac{1}{2^{1+\ell}}$
- $(iii)$ $k$ is such that $\varepsilon_0 \cdot k \ge 2t$
- $(iv)$ we choose $k_\ell$ inductively on $\ell \le k$ such that $k_\ell$ large enough
- $(v)$ $c_1$ a large enough real
- $(vi)$ $n_0$ is large enough.

For a node $x$ of the circuit $C$ let $\mathscr{Y}_x$ be the set of nodes which fans into it (i.e. its sons; without loss of generality in the level 1 we have only OR).

Given a circuit $C$ and a restriction $\rho$, we can compute the value given by $\rho$ to some $\sigma$ of the nodes of $C$. Formally we define by induction on the level of the node:

- $(vii)$ let $x$ be a node of $C$ of level 0, of the form $z_i$ (respectively $\neg z_i$). Then if $i \in \mathrm{Dom}(\rho)$ we say that $\rho$ computes the value of $x$ and that the value is $\rho(i)$ (respectively $1 - \rho(i)$)
- $(viii)$ let $x$ be a node of $C$ of level $\ell+1$, of the form OR. Then if for some $y \in Y_x, \rho$ computes the value of $y$ and the value is 1 we say $\rho$ computes the value of $x$ and the value is 1. If for all $y \in Y_x, \rho$ computes the value of $y$ and the value is 0 we say $\rho$ computes the value of $x$ and the value is 0. If none of the above holds we say $\rho$ does not compute the value of $x$.

We deal similarly with nodes of form AND. With the above definition w consider $C\restriction\rho$ as a circuit on the variables $\{z_i|\rho(i)-x\}$, the nodes of $C|\rho$ are the nodes of $C$ the value of which is not computed by $\rho$.

First we assume $d > 2$.

Note

$\otimes_1$ drawing as below a balanced restriction $\rho$ with domain with $\leq n$ elements, with probability $\geq 1-\varepsilon/3$ we have: in $C^1 = C\restriction_\rho$, every node of the level 1 (i.e. for which $\mathscr{Y}_x$ is a set of atoms) satisfies $|\mathscr{Y}_x| \leq c_0(lnn + c'_0)$.

[Why? Choose randomly a set $\mathbf{u}_0$ of $[n/2]$ pairwise disjoint pairs of numbers among $\{1,\dots,2n+1\}$, and then for each $\{i,j\} \in \mathbf{u}_0$ decide with probability half that $\rho(i)=0$, $\rho(j)=1$ and with probability half that $\rho(i)=1$, $\rho(j)=0$ (independently for disjoint pairs). This certainly gives a balanced $\rho$.

Now if $x$ is a node of $C$ of the level 1, the probability that $\rho$ does not decide the truth value which the node compute is $\leq (\frac{1}{2})^{|\mathscr{Y}_x|}$. Note: after drawing $\mathbf{u}$, if $\mathscr{Y}_x$ contains a pair from $\mathbf{u}$ the probability is zero, we only increase compared to drawing just a restriction. So the probability that for *some* $x$ of the level 1 of $C$, $|\mathscr{Y}_x| \geq (\ln 4)t(\ln n)+1$ and the truth value is not computed, is $\leq |C| \times (\frac{1}{2})^{(ln2)t(\ell nn)+1} \leq \frac{\varepsilon}{3}$, so with probability $\geq 1-\frac{\varepsilon}{3}$ for any such $x$ the truth value is computed.]

Next, we say that a restriction $\rho'$ extends a restriction $\rho$ if $\rho'(i) \neq \rho(i) \Rightarrow \rho(i) = *$.

Now

$\otimes_2$ choosing randomly a restriction $\rho_1$ as below we have: $\rho_1$ is a balanced restriction extending $\rho_0$ such that $|\{i : i \in \{1,\dots,2n+1\}, \rho_0(i) = *\}| \geq 2[n^{\varepsilon_0}]+1$ and with probability $\geq 1-\varepsilon/3$ in $C^1 = C\restriction\rho_1$ for every node $y$ of the level 1 we have, $|\mathscr{Y}_y| \leq k$.

[Why? We draw a set $\mathbf{u}_1$ of $(2n+1-|\mathrm{dom}(\rho_0)| - (2[n^{\varepsilon_0}]+1))/2$ pairs from $\{i : \rho_0(i) = *\}$ pairwise disjoint and for each $\{i,j\} \in \mathbf{u}_0$, decide with probability $\frac{1}{2}$ that $\rho_1(i)=0$, $\rho_1(j)=1$ and with probability half that $\rho_1(i)=1$, $\rho_1(j)=0$.

For each node $y \in C^1$ of the level 1 by $\otimes_1$ we may assume $|cY_y| \leq c_0\ell nn + c'_0$ and the probability that "the number of $y' \in \mathscr{Y}_y$ not assigned a truth value by $\rho_1$ is $\geq k+1$" is at most $(\binom{|\mathscr{Y}_y|}{k+1}) \times \left(\frac{1}{2n^{\varepsilon_0}+1}\right)^{k+1} \leq (c_0 \ln n + c'_0)^{k+1} \cdot n^{-\varepsilon_0(k+1)} < n^{-t} \cdot \frac{\varepsilon}{3}$.]

We now choose by induction on $\ell \leq k$ a restriction $\rho_{2,\ell}$ such that

$\otimes_3$ $(a)$ $\rho_{2,\ell_0} = \rho_1$, $\rho_{2,\ell} \subseteq \rho_{2,\ell+1}$, $2n+1-(2[n^{\varepsilon_\ell}]+1) = |\mathrm{dom}\rho_{2,\ell}|$

$(b)$ every $y \in C$ of the level 2 there is a set $w_{y,\ell}$ of $\leq k_\ell$ atoms such that: if $z \in \mathscr{Y}_y$, then $|\mathscr{Y}_z \setminus w_{y,\ell}| \leq k-\ell$.

Now for $C \restriction \rho_{2,k}$ we can invert AND and OR in the first and s econd levels (multiplying the size by a constant $\leq c_1$) decreasing $d$ by one thus carrying the induction step.

For $\ell = 0$ let $\rho_{2,0} = \rho_1$. For $\ell+1$, for each $y \in C$ of level 2 let $\Xi = \{\nu : \nu$ a restriction with domain $w_{y,\ell}\}$ let

$$\mathscr{Y}_y^\nu = \{z \in \mathscr{Y}_y : \text{ the truth value at } z \text{ is still not computed under } \rho_{2,\ell} \cup \nu\},$$

and for $z \in \mathscr{Y}_y^0$ let $\mathrm{dom}(z) = \{x \in \mathscr{Y}_z : x \notin \mathrm{Dom}(\rho_{2,\ell} \cup D)$.

Now try to choose by induction on $i$ in node $z_{y,\ell,\nu,i} \in \mathscr{Y}^\nu_y \setminus \{z_{y,\ell,\nu,j} : j < i\}$, such that $\mathrm{dom}(z_{y,\ell,\nu,i})$ is disjoint to $\bigcup\limits_{j<i} \mathrm{dom}(z_{y,\ell,\nu,j}) \setminus w_{y,\ell}$. Let it be defined if $i < i_{y,\ell}$.

Now $\rho_{2,\ell+1}$ will for each $\nu \in \Xi$ decide that $\nu$ make the truth value computed in $y$ true, or will leave only $\leq (k_{\ell+1} - k_\ell)/2^{k_\ell}$ of the atoms in $\bigcup\limits_i \mathrm{dom} z_{y,\ell,\nu,i} \setminus w_{y,\ell}$ undetermined (this is done as in the previous two stages).

But now by $\otimes_1 + \otimes_2$, $C \restriction_{\rho_{2,k}}$ can be considered having $d-1$ levels (because, as said above we can invert the AND and OR in level 1 and 2).

We have translate our problem to one with $[n^{\varepsilon_k}]$, $d-1$, $\varepsilon_k(t+\varepsilon_1)$, $\frac{\varepsilon}{3}$ instead $n$, $d$, $t$, $\varepsilon$ (the $t+\varepsilon$ is just for $n^{t+\varepsilon} > c_1 n^t$).

Also note: $\varepsilon_k, c_1$ does not depend on $n$. So we can use the induction hypothesis. We still have to check the case $d \leq 2$, we still are assuming level 1 consist of cases of OR, and for almost all random $\rho_0$ (as in $\otimes_1$) for every $x$ of level 1 we have $|\mathscr{Y}_x| \leq \mathbf{c}_0 lnn + c'_0$ (so again changing $n$).

So as above we can add this assumption. Choose randomly a complete restriction $\rho^0$ with $|\{i : \rho^0(i) = 1\}| = n$, and let $\rho^1$ be gotten from $\rho^0$ by changing one zero to 1, so $|\{i : \rho^1(i) = 1\}| = n+1$.

Now the probability that $C \restriction_{\rho^0} = 0$ but $C \restriction_{\rho^1} = 1$ is small: it require that for some node $x$ of level 1 is made false in $C \restriction_{\rho^0}$ while there is no such $x$ for $C \restriction_{\rho^1}$, but if $x(*)$ is such for $C \restriction_{\rho^0}$ it is made true then with probability $\geq 1 - \frac{|\mathscr{Y}_x|}{2n+1} \geq 1 - \frac{\mathbf{c}_0 lnn}{n}$ the $z_i$ changed is not in $\mathscr{Y}_{x(*)}$. Contradiction, thus finishing the proof.

## § 4. Two Place Functions

Here we consider the random structure $([n], F_n)$ where $F_n(x, y)$ is a random function from $[n] \times [n]$ to $[n]$. (We no longer have order. A typical sentence would be $\forall_x \exists_y F(x, y) = x$.): Again for any sentence $A$ we define $f(n) = f_A(n)$ to be the probability $A$ holds in the space of structures on $[n]$ with uniform distribution. Again it is known [CHSh:245] that convergence fails, there are $A$ for which $f(n)$ does not converge. Again our result is a very weak zero one law.

{0.4}
**Theorem 4.1.** *For every* $A$

$$\lim_{n\to\infty} f_A(n+1) - f_A(n) = 0.$$

Again let $m = 2n+1$. Let $F^*(x, y, z)$ be a *three*-place function from $[m]\times[m]\times[m]$ to $[m]$. For $S \subset [m]$ of cardinality $i = n$ or $n + 1$ we define $F^*_S$, a partial function from $[S] \times [S]$ to $[S]$ by setting $F^*_S(x, y) = F^*(x, y, z)$ where $z$ is the minimal value for which $F^*(x, y, z) \in S$. If there is no such $z$ then $F^*_S(x, y)$ is not defined. This occurs with probability $(\frac{m-i}{m})^m$ for any particular $x, y$ so the probability $F^*_S$ is not always defined is at most $i^2(\frac{m-i}{m})^m = o(1)$.

We generate a random three-place $F^*$ and then consider $F^*_S$ with $S$ a random set of size $i = n$ or $n + 1$. Conditioning on $F^*_S$ being always defined it then has the distribution of a random two-place function on $i$ points. Thus $\Pr[A]$ over $[n], F_n$ is within $o(1)$ of $\Pr[A]$ when $F_n = F^*_S$ is chosen in this manner. Thus, as in §2, it suffices to show for any $F^*$ and $A$ that, letting $g(i)$ denote the probability $F^*_S$ satisfies $A$ with $S$ a uniformly chosen $i$-set, $g(n + 1) - g(n) = o(1)$. Again fix $F^*$ and $A$ and let $z_x$ be the Boolean value of $x \in S$ for $1 \le x \le 2n + 1$. In $A$ replace the ternary relation $F(a, b) = c$ by $\bigwedge_{y<c} \neg z_{F^*(a,b,y)} and z_{F^*(a,b,c)}$. (For $c = 1$ the left hand part is simply True.)

As in §2 replace $\exists_x P(x)$ by $\bigvee_x (z_x \wedge P^*(x))$ where $P^*(x)$ has been inductively defined as the replacement of $P(x)$. Then the statement that $F^*_S$ satisfies $A$ becomes a Boolean function of the $z_1, \dots, z_m$, as before it is an $AC^0$ function, and by §2 we have $g(n + 1) - g(n) = o(1)$.

* * *

The following discussion is directed mainly for logicians but may be of interest for CS-oriented readers as well.

{1.6}
**Discussion 4.2.** 1) Note that the results of [?] cannot be gotten in this way as the proof here use high symmetry.

The problem there was: let $\bar{p} = \langle p_i : i \in \mathbb{N}\rangle$ be a sequence of probabilities such that $\sum_i p_i < \infty$. Let $G(n, \bar{p})$ be the random graph with set of nodes $[n] = \{1, \dots, n\}$ and the edges drawn independently, and for $i \ne j$ the probability of $\{i, j\}$ being an edge is $p_{|i-j|}$.

The very weak 0-1 law was proved for this context in [?] (earlier on this context (probability depending on distance) was introduced and investigated in Łuczak and Shelah [LuSh:435]). Now drawing $G(2n + 1, \bar{p})$ and then restricting ourselves to a random $S \subseteq \{1, \dots, 2n+1\}$ with $n$, and with $n+1$ elements, fail as $G(2n+1, \bar{p}) \restriction_S$ does not have the same distribution as $G(|S|, \bar{p})$.

2) We may want to phrase the result generally;

One way: just say that $M_n, M_{n+1}$ can be gotten as above : draw a model on $[2n+1] = \{1, \ldots, 2n+1\}$ (i.e. with this universe), then choose randomly subsets $P^\ell_n$ with $n + \ell$ elements and restrict yourself to it

3) Two random linear order satisfies the very weak 0-1 zero law (mean: take two random functions from $[n]$ to $[0,1]_{\mathbb{R}}$). The proof should be clear.
4) All this is for fixed probabilities; we then can allow probabilities depending on $n$ e.g. we may consider $G_<(n, p_n)$ is the model with set of elements $\{1, \ldots, n\}$, the order relation and we draw edges with edge probability $p_n$ depending on $n$. This call for estimating two number (for $\varphi$ first order sentence):

$$\alpha_n = |\text{Prob}(G_<(n, p_{n+1}) \models \varphi) - \text{Prob}(G_<(n, p_n) \models \varphi)|$$

$$\beta_n = |\text{Prob}(G_<(n, p_{n+1}) \models \varphi) - \text{Prob}(G_<(n+1, p_{n+1} \models \varphi)|$$

As for $\beta_n$ the question is how much does the proof here depend on having the probability $\frac{1}{2}$. Direct inspection on the proof show it does not at all (this just influence on determining the specific Boolean function with 2n+1 variables) so we know that $\beta_n$ converge to zero.

As for $\alpha_n$, clearly the question is how fast $p_n$ change.
5) As said in [Sh:463] we can also consider $\lim\big(\text{Prob}_{n+h(n)}(M_{n+h(n)} \models \psi) - \text{Prob}_n(M_n \models \psi)\big) = 0$, i.e. characterize the function $h$ for which this holds but this was not dealt with there. Hopefully there is a threshold phenomena. Probably this family of problems will appeal to mathematicians with an analytic background.

Another problem, closer to my heart, is to understand the model theory: in some sense first order formulas cannot express too much, but can we find a more direct statement fulfilling this?

* * *

Another way to present the first problem for our case is: close (or at least narrow) the analytic gap between [?] and the present paper.

After this work, Boppana and Spencer [BS82], continuing the present paper and [CHSh:245], address the problem and completely solve it. More specifically they proved the following.

> For every sentence $A$ there exists a number $t$ so that $m(n) = O(n \ln^{-t} n)$ implies
>
> $$\lim_{n\to\infty} f_A(n + m(n)) - f_A(n) = 0.$$

And

> For every number $t$ there exists a sentence $A$ and a function $m(n) = O(n \ln^{-t} n)$ so that $f_A(n+m(n)) - f_A(n)$ does not approach zero.

Together we could say: a function $m(n)$ has the property that for all $A$ and all $m'(n) \leq m(n)$ we have $f_A(n+m'(n)) - f_A(n) \to 0$ if and only if $m(n) = o(n \ln^{-t} n)$ for all $t$.

For improving the bound from this side they have used Hastad switching lemma [Hastad] (see [AS], §11(2), Lemma ??).

6) If we use logic stronger than first order , it cannot be too strong (on monadic logic see [KfSh:201]), but we may allow quantification over subsets of size $k_n$, e.g. $\log(n)$ there are two issues:

- $(A)$ when for both $n$ and $n+1$ we quantify over subsets of size $k_n$, we should just increase $M$ by having the set $[n]^{k_n}$ as a set of extra elements, so in (*), $P$ is chosen as a random subset of $\{1, 2, 3, \ldots, 2n, 2n+1\}$ with $n$ or $n+1$ elements but the model has about $(2n+1)^{k_n}$ elements; this require stronger theorem, still true (up to very near to exponentiation)
- $(B)$ if $k_n \neq k_{n+1}$ we need to show it does not matter, we may choose to round $k_n = \log_2(n)$ so only for rare $n$ the value change so we weaken a little the theorem *or* we may look at sentences for which this does not matter .

  Maybe more naturally, together with choosing randomly $\mathscr{M}_n$ we choose a number $\underline{k}_n$, and the probability of $\underline{k}_n = k_n + i$ if $i \in [-k_n/2, k_n/2]$ being $1/k_n$.

  And we ask for "$p_n^\varphi =$: Prob($\mathscr{M}_n \models \varphi$ where the monadic quantifier is interpreted as varying on set with $\leq \underline{k}_n$ elements) for sentence $\varphi$ (the point of the distribution of $\underline{k}_n$ is just that for $n$, $n+1$ they differ a little). E.g. if for a random graph on $n$ (probability **??**) we ask on the property "the size of maximal clique of size at most $[\log_2 n]^2$ is even" it satisfies the very weak zero one law

Of course we know much more on this, still it shows that this old result (more exactly - a weakened version) can be put in our framework.

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