(E35)A BOTHERSOME QUESTION

Q-Mathematics, mathematics, has everything not been discovered already generations ago?

A-What an educated person knows today generally was discovered hundreds of years ago, for the most part he doesn't know not only what was discovered, but also what questions mathematicians are asking in our century, and it's a pity. It seems like the more mathematics has advanced, deepened and become beautiful, and its use has broadened and wonderful theories and basic puzzling questions have been answered, it has become unknown and closed to those outside of a select few. Phycisists can assume that the reader has heard about black holes, and biologists - aobut DNA, a mathematician would be luck if the reader know something about the ? of Liebnitz and Newton from the 17th cent.

Q - What can be "beautiful" about mathematics?

A - Michaelangelo said that he just discovers the form hidden in the marble and I can identify with him.

Q - Perhaps you can explain what you hope to discover?

A - I'll try to explain without cheating too much. A problem I love and that has been bothering me for many years, that many have dealt with: what are the arithmetic laws of infinite numbers.

Q - What are these numbers, who discovered them?

A - These are numbers which measure the number of elements in infinite sets. Usually when the question is asked: how many members/elements are there in a set, the notion of the number is already clear, which isn't the case in infinite sets.

Q - What do you mean, isn't there only one infinite number – infinity!

A - This has no meaning if we have not defined the infinite numbers. We can know when, in two sets, there is the same number of elements even if we haven't counted them, such as, for instance the number of living people, which equals the number of brains of living people (if the reader will ignore several politicians who get on his nerves). In general, two sets, say the Tribe of Simeon and the Tribe of Levi will be considered as having the same number of elements if one can find a "matching" between the Tribe of Simeon and the Tribe of Levi – to each Simeonite will be matched one single Levite and vice versa. Thus numbers can be defined, including the infinite numbers. Cantor, towards the end of the 19th century, discovered them and named number of natural numbers (i.e. 0, 1, 2, ..., n, ...) $\aleph_0$.

Q - What can one do with them?

A - It turns out that you can naturally define arithmetic operations: addition, multiplication and power (x to the power of y) (among numbers, finite and infinite).

Q - Can you explain what these operations are? For instance, addition?

A - In giving 2 disjoint sets (in other words, without a common element), the number of elements in union will be the sum of the number of elements. It seems that $\aleph_0 = \aleph_0 + 2$, because the number of natural numbers (0, 1, 2, 3, ...)

equals the number of natural numbers greater than 1 (i.e. 2, 3, 4, 5, ..), why? Because one can "match" 0 to 2, 1 to 3 and in general, n to $n+2$.

Q - Doesn't this show that everthing is nonsense? It negates Aristotle's great rule "the whole is greater than the part", and therefore for all numbers a, b greater than 0, $a+b>a$.

A - It's an exaggeration to expect that all usual rules of arithmetic will continue to exist, what's so beautiful and surpr ising is that many of them DO exist: all the rules that are equalities? For instance $(a+b)c = ac+bc$ and $(a^b)^c = a^{bc}$ (and for the pedantic: for every numbers, a, b, we have $a=b$ or $a<b$ or $b<a$.

Q - So surely those operations are very awkward and it's impossible to know anything about these numbers.

A - To the contrary, in a certain way this arithmetic is more transparent. It was discovered that the sum of two numbers where at least one of them in infinite is the bigger number of the two, and it is similar of the product of non-zero number. If this were true for finite numbers, it would mean that 7 + 123 = 123. Wouldn't you prefer to make computations such as this in school?

Q - If so, it's all too simple, and in effect there is actually only one infinite number that is necessarily $\aleph_0$, which would solve all the problems.

A - The number of natural numbers is not equal to the number of real numbers (in other words, infinite decimal fractions) or what is equivalent, the number of points in the plane. This can be seen in a particular case: For every number a, "$2^a > a$". Also, for every number a there is a successor, the smallest number bigger than itself, which we will call a^+, so we can define $\aleph_1$ as the successor of $\aleph_0$, number following $\aleph_0$; $\aleph_2$ as the successor of $\aleph_1$ and we can continue to define $\aleph_n$ for any natural number n.

Q - And that's it?

A - No, for example, for every n there exists the first number under which there are $\aleph_n$ numbers and it is called ($\aleph_{\aleph n}$.

Q - If addition and multiplication are so simple, then probably the power operation is not so complicated.

A - As you recall, we have two functions which increase the (infinite) number: the successor a^+ and the power 2^a. It's extremely tempting to hope that they are actually one operation, i.e. for every infinite a $2^a = a^+$ this hypothesis is called the Generalized Continuum Hypothesis. If this hypothesis is correct, then the power operation is very simple and we would completely understand laws of arthmetic of infinite numbers.

Q - Of what use would this solution be?

A - Whoever wants to prove general theorems on "large" infinite sets, this hypothesis will be very useful.

Q - Do mathematicians truly consider this an important problem?

A - When Hilbert, considered the outstanding mathematician since the beginning of the 20th century, prepared a list of the most important mathematical problems, with 23 questions (this is the best-known list of its kind), he chose this as question number 1 (he asked: is $\aleph_1 = 2^{\aleph 0}$, but he meant: find all the arithmetic laws for infinite numbers). On the other hand, the majority of mathematicians whom I have met aren't particularly interested in it, all mathematicians are in agreement among themselves as to what is correct, but not necessarily about what is important, and what is beautiful and exciting. Mathematics is an Exact Art.

Q - Presumably, do you hope to prove the Generalized Continuum Hypothesis? or at least to refute it? Or have you missed the boat?

A - Too late. Godel showed that (from the usual axioms of set theory) it's impossible to contradict it. On the other hand, it was proved that it's impossible to prove it, moreover besides $2^{\aleph 0}$ not greater than $2^{\aleph 1}$, $2^{\aleph 1}$ not greater than $2^{\aleph 2}$ etc. (and a little more) there aren't actually any additional restrictions.

Q - If this is so, then there is nothing new to discover?

A - No, because if you look at products of "few" large numbers (for instance, just $\aleph_0$ numbers), there is a lot to say. For instance, if $2^{\aleph_0}$ is any $\aleph_n$, then the product of all the $\aleph_n$'s is not large, it is smaller than $\aleph_{\aleph 4}$.

Q - Isn't there a typographical error here ? It seems to me that you are talking about infinite nunmber, so why in the dickens does 4 appear here?

A - That's exactly what I want to find out, and I feel like it's a a basic problem, the key to the mystery of the mathematical laws of infinite numbers.