Theorem. If a, b and c are r.e. degrees, a is incomparable to b, c is the infimum of a and b, and c is cappable; then a and b, each, is cappable.

Corollary. The seven element lattice known as the double diamond lattice cannot be embedded in the r.e. degrees so as to preserve least and greatest element.

With work done by Ambos-Spies in his Ph.d thesis and the above theorem, the distributive lattices with 0 and 1 that can be embedded in the r.e. degrees (preserving 0 and 1) are completely characterized.
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86T-03-268 G. Sageev, N. Shumish and S. shelah. State University of N.Y. and the Hebrew University, There are Noetherian domain in every power with free additive group II

Claim: If $Y \cup \{x(1),\ldots,x(n)\} \subseteq X$, $x(\ell) \in X - Y$ distinct, $G = R^+_{W \cup Y}$, $I = \Sigma_i I_i$, $I_i = R^+_{W(i) \cup Y}$ then G/I is free, when Y is countable. Pf: It suffices: (a) G/I is torsion free. (b) if $a_1 \ldots, a_k \in G/I$ are independent then $\{m \in Z^+ :$ there are $\langle q_1 \ldots, q_k \rangle \in L$ s.t. $\Sigma q_i a_i$ is divisible by m in G/I$\}$ is finite, where $L = \{\langle q_1,\ldots,q_k \rangle : q_i \in Z$, not all zero with no common divisor$\}$. Let $x_1(q) \in X$ new distinct variable $V = \{x_1(1),\ldots,x_1(n)\}$. For $u \subseteq \{1,\ldots,n\}$, $h_u: R_{W \cup Y} \to R_{V \cup Y}$ an isomorphism, $h_u(y) = y$ for $y \in Y$, $h_u(x(q)) = x_1(q)$ if $q \in u$, $h_u(x(q)) = x\ (q)$ if $q \notin u$. So let $a_1 + I,\ldots, a_k + I$ be independent. Suppose $\langle q_1,\ldots,q_k \rangle \in L$, $m_0 m_1 \in Z - \{0\}$, $m_0 m_1$ divides $\Sigma_i m_0 m_1 q_i a_i + I$. So for some $s \in R_{W \cup Y}$, $p_i \in I_i$: $\Sigma_i m_0 q_i a_i = m_0 m_1 s + \Sigma_i p_i$. Let u vary on subsets of $\{1,\ldots,n\}$, $b_i = \sum_u (-1)^{|u|} h_u(a_i)$, so $\sum_i m_0 q_i b_i = \sum_u (\Sigma_i m_0 q_i (h_u(a_i)) = m_0 m_1 \Sigma_u h_u(s) + \Sigma_i \Sigma_u h_u(p_i)$. However $\sum_u h_u(p_i)$ is zero (as x(i) does not appear in it). So $\Sigma_i \sum_u m_0 q_i h_u(a_i) = m_0 m_1 \sum_u h_u(s)$. As $R^+_{Y \cup W \cup Y}$ is free, it suffices to prove $\{b_i : i=1,n\}$ is independent, equivantely they are linearly independent (over the rationals) in $Z(Y \cup W \cup V)$. But we can substitute suitable numbers for $x_1(1),\ldots,x_1(n)$ and get cont. to "$\{a_i + I : i = 1,\ldots,n\}$ is independent". (Received June 30, 1986)

86T-03-269 G. Sageev and S. Shelah, State University of N.Y. at Buffalo and the Hebrew University, Jerusalem. There are Noetherian domain in every cardinately with free additive group I.

Th. There are Noetherian rings (in fact domains) with a free additive group, in every infinite cardinality . Remark 1) For $\aleph_1$ this was proved by [1] O'Neill 2) The work was done in Sept. 83. 3) We thank Fuchs for suggesting to us the problem Sketch of pf: Z the ring of integers, X a set of distinct variables, Z[X] the ring of polinomials over Z, Z(X) its field of quotients, and $R_X = \{p/q : p \in Z[X], q \in X$, p not divisible (non trivially) by any integer$\}$ $\subseteq Z(X)$. As in [1] R is a Noetherian domain. Let for a ring R, R^+ be its additive group. For $Y \subseteq X$ we can define Z[Y], Z(y), R_Y similarly. Lemma 1) R^+ is a free abelian group. 2) if $n \geq 0, Y \subseteq X$, $x(1),\ldots,x(n) \in X-Y$ distinct, $W = \{x(1),\ldots,x(n)\}$, $W(\ell) = W - \{x(\ell)\}$ then $R^+_{W \cup Y} / \sum_{\ell=1}^{n} R^+_{W(\ell) \cup Y}$ is free. Pf: 1) Follows by (2) for n = 0, Y = X 2) This is phrased because it is the natural way to prove (1) by induction on $|Y|$, for all n simultaneously. If $|Y| > \aleph_0$, let $Y = \{y(\alpha) : \alpha < \lambda\}$, $\lambda = |Y|$, $Y_\alpha = \{y(i) : i < \alpha\}$. It suffices to prove $R^+_{Y_{\alpha+1}} / (\sum_{\ell=1}^{n} R^+_{Y_{\alpha+1} \cup W(\ell)} + R^+_{Y_\alpha \cup W})$ (which is isomorphic to $(R^+_{Y_{\alpha+1}} + \sum_{\ell=1}^{n} R^+_{Y \cup W(\ell)}) / (\sum_{\ell=1}^{n} R^+_{Y_{\alpha+1} \cup W(\ell)} + R^+_{Y_\alpha \cup W}))$ is free. Use induction Hypothesis the next abstract complete the case "y countable". (Received June 30, 1986)

86T-03-270 MICHAEL DEUTSCH, Department of Math., Univ. of Bremen, West-Germany. Eine Bemerkung zur spektralen Darstellung von Prädikaten durch Ausdrücke aus $\exists^\infty \forall^3 \exists(\varrho,1)$, $\forall^\infty \exists(\varrho,1)$ und $\forall^3 \exists(\infty,1)$.

For any ϱ-ary enumerable predicate P and any ϱ-ary coenumerable predicate Q with $Px_1 \ldots x_\varrho \rightarrow Qx_1 \ldots x_\varrho$ one can find formulas $\beta_1 \in \exists^\infty \forall^3 \exists(\varrho,1)$, $\beta_2 \in \forall^\infty \exists(\varrho,1)$ and $\beta_3 \in \forall^3 \exists(\infty,1)$ containing the set variables $P_1,\ldots,P_\varrho$ and the binary pred. variable E such that $(1 \leq k \leq 3)$: $Qx_1 \ldots x_\varrho \Leftrightarrow$ There is a reduced model I of β_k with $|I(P_\varkappa)| = x_\varkappa$ $(1 \leq \varkappa \leq \varrho) \Leftrightarrow \beta_k$ is satisfiable on a transitive domain of sets by an interpretation I with $I(E) = \in_k \wedge I(P_\varkappa) = \{x \mid x < x_\varkappa\}$ $(1 \leq \varkappa \leq \varrho)$.

$Px_1 \ldots x_\varrho \Leftrightarrow$ There is a reduced finite model I of β_k with $|I(P_\varkappa)| = x_\varkappa$ $(1 \leq \varkappa \leq \varrho) \Leftrightarrow$ β_k is satisfiable on a finite transitive domain of sets by an interpretation I with $I(E) = \in_k \wedge I(P_\varkappa) = \{x \mid x < x_\varkappa\}$ $(1 \leq \varkappa \leq \varrho)$.