Recall: $\square_\lambda$ asserts the existence of a seq. $\langle C_\alpha : \alpha<\lambda^+\rangle$ s.t. C_α is a club in α, $otp(C_\alpha) < \lambda$ and if β is a limit point of C_α then $C_\beta = C_\alpha \cap \beta$.

$\square'_\lambda$ asserts the existence of a seq. $\langle A_\alpha : \alpha<\lambda^+\rangle$ s.t. $A_\alpha \subseteq P(\alpha)$ $|A_\alpha| \le \lambda$ and for every α there is $C_\alpha \in A_\alpha$, C_α a club in α $otp(C_\alpha) < \lambda$ and for any limit point β of C_α, $C_\alpha \cap \beta \in A_\beta$.

Our theorem answers a question raised by Jensen in : R.Jensen, "The fine structure of the constructible hierarchy". Annals of Math. Logic, 4 [1972]. (Received December 5, 1983)

*84T-03-154 SHAI BEN-DAVID and SAHARON SHELAH, Hebrew University of Jerusalem, Givat Ram 91904, Jerusalem, Israel. Non-Special Ahronszajn trees on Successors of Singulars.

When one tries to prove the consistency of "ZFC + GCH + there are no $\aleph_{\omega+1}$ Souslin trees" it may be natural to try to specialize all $\aleph_{\omega+1}$ Ahronszajn trees. We show that such an attempt is bound to fail (as long as there are Ahronszajn trees in the model). It follows that the situation regarding Ahronszajn and Souslin trees at λ^+ is different for a regular λ than it is for a singular λ. Let λ denote a strong limit singular cardinal s.t. $2^\lambda=\lambda^+$. Thm.1 (Shelah): If there is a λ^+-Ahr. tree then there is a non-special one. Call a tree '(κ,∞)-distributive', if the intersection of any $<\kappa$ dense-open subsets of the tree is a dense open set. For a λ^+-Ahr. tree T, T is Souslin$\to$T is (λ^+,∞) distributive$\to$T is not special. Thm.2 (Ben-David). If there is a λ^+-Special Ahr. tree, then there is a '(λ^+,∞)-distributive' λ^+-Ahr. tree. Thm.3 (Ben-David). If $cof(\lambda)>\omega$ and there is a λ^+-Ahr. tree, then, the existence of a '$(cof(\lambda)^+,\infty)$ distributive' λ^+-Ahr. tree, follows from the principle "Every stationary subset of $\{i<cof(\lambda)^+ : cof(i)<cof(\lambda)\}$ has an initial segment stationary in its supremum". Conclusion: G.C.H. $\lambda=\aleph_{\omega_1}$. If there are no $\aleph_2$-Souslin trees, and there is an λ^+-Ahr. tree then there is a $(\aleph_2,\infty)$ distributive λ^+-Ahr. tree. A similar conclusion holds if we replace ω_1 by any uncountable regular cardinal.
(Received December 5, 1983)

*84T-03-158 LUC BÉLAIR, Yale University, Box 2155, Yale Station, New Haven, Connecticut 06520. The Universal Part of the Theory of p-Adically Closed Fields (pCF). Preliminary report.

Macintyre showed that pCF admits elimination of quantifiers in the language L of valued fields augmented with 1-ary predicates P_n to denote n-th powers. We have found an explicit axiomatisation T for the universal part of pCF in L. The key axiom (scheme) ensures that if $P_n(x)$ then the P_n-type of an eventual n-th root of x is carried by T.

Our proof gives no unique way to go from a model of T to a p-adically closed field. The following is proved. Proposition 1. If $\mathfrak{A} \models T$ and B is an immediate henselian extension field of A then B can be expanded to a model $\mathfrak{B} \supseteq \mathfrak{A}$ of T. Using T we are able to give an analysis of pCF in L "a la Robinson", i.e. show that pCF is the model-completion of T, in parallel with his treatment of real-closed fields. (Received December 7, 1983)

84T-03-160 Saharon Shelah and Ani Loa-Khlum, The Hebrew University, Jerusalem, Israel. An $\aleph_2$- Souslin tree from a strange hypothesis.

Theorem: Suppose CH holds and the filter $D(\omega_1)$ (see below) is $\aleph_2$-saturated. Then there is an $\aleph_2$-Souslin tree. Notation. $D(\omega_1)$ is the filter generated by the closed unbounded subset of ω_1. Let $S^\alpha_\beta = \{\delta<\aleph_\alpha : cf\ \delta = \aleph_\beta\}$. Proof: It is known that the assumption implies $2^{\aleph_1}=\aleph_2$. By Gregory, if there is a stationary $S\subseteq S^2_0$ with no initial segment stationary, then there is an $\aleph_2$-Souslin tree, so we assume there is no such S. By Gregory $\diamondsuit(S^2_0)$ holds, and let $\langle A_\delta : \delta \in S^2_0\rangle$ exemplify this. For $\delta \in S^2_1$ define $P_\alpha = \{B\subseteq\alpha : \{\delta<\alpha : B\cap\delta = A_\delta\}$ is a stationary subset of $\alpha\}$. If $|P_\alpha|>\aleph_1$ let $B_i \in P_\alpha$ $(i<\aleph_2)$ be distinct $\langle\gamma(\zeta):\zeta<\omega_1\rangle$ increasing continuous $\gamma(\zeta)<\alpha$, $\bigcup_\zeta \gamma(\zeta) = \alpha$; let $S_i \overset{df}{=} \{\zeta<\omega_1 : B_i \cap \gamma(\zeta)=A_{\gamma(\zeta)}\}$ S_i is stationary (as $B_i \in P_\alpha$) and $S_i \cap S_j$ is bounded for $i \ne j$ (as for some $\zeta_0<\omega_1$, $B_i \cap \gamma(\zeta_0)\ne B_j\cap\gamma(\zeta_0)$ hence $S_i\cap S_j\subseteq\zeta_0$).

But $D(\omega_1)$ is $\aleph_2$-saturated, contr., hence $|P_\alpha| \le \aleph_1$. Also for every $A \subseteq \omega_2$, $\{\delta \in S_0^2 : A \cap \delta = A_\delta\}$ is stationary hence for some $\alpha \in S_1^2$, $\{\delta \in S_0^2 \cap \alpha : A \cap \delta = A_\delta\}$ is stationary below α. So $\langle P_\alpha : \alpha \in S_1^2 \rangle$ exemplify a variant of $\Diamond$ which by Kunen implies $\Diamond(S_1^2)$; together with $2^{\aleph_0} = \aleph_1$, $2^{\aleph_1} = \aleph_2$ we finish. Remark: We can replace $\aleph_0$, $D(\omega_1)$ by any regular $\aleph_\alpha$ and $D(\aleph_{\alpha+1}) + S_\alpha^{\alpha+1}$. (Received December 7, 1983)

*84T-03-163 MARIAN BOYKAN POUR-EL and IAN RICHARDS, University of Minnesota, Minneapolis, Minnesota 55455 On the Computability of Eigenvectors of Effectively Determined Self-adjoint Operators

In a previous paper (to appear in Advances in Math.), the authors showed that the eigenvalues of an effectively determined, bounded or unbounded, self-adjoint operator are computable. Here we show that the corresponding eigenvectors need not be computable. More precisely: There exists an effectively determined, compact, self-adjoint operator T such that (a) 0 is an eigenvalue of T of multiplicity one, and (b) no eigenvector corresponding to 0 is computable. (Received December 10, 1983)

*84T-03-166 JOHN VAUGHN, University of Illinois at Chicago, Chicago, Illinois 60607. The Ideal of Forking Formulas in a Stable Theory.

Let M be the monster model for a complete stable theory. Let B(M) be the Lindenbaum algebra of n-ary formulas with parameters from M. $A \subseteq M$. $F_A = \{\varphi(\bar{x},\bar{m}) : \varphi \text{ forks over } A\}$. $A_A = \{\varphi(\bar{x},\bar{m}) : \varphi \text{ is almost over } A\}$.

1) F_A is an ideal in B(M). Define $\varphi \sim \psi$ iff $\neg[\varphi \leftrightarrow \psi] \in F_A$.

The finite equivalence relation theorem can be reformulated as:

2) $B(M)/\sim \;\simeq A_A$. This gives a strong normalization result:

3) φ is normal w.r.t. $\sim$ iff φ is almost over A.

4) $B(M) \simeq A_A \times F_A$ (as Boolean groups). (Received December 16, 1983)

*84T-03-167 RAMI GROSSBERG, Institute of Mathematics, The Hebrew University, Jerusalem, Israel. Chang Shelah two cardinal theorem for successors of singulars.

Theorem. Let λ be singular such that $\square_\lambda$ holds, and there exists an unbounded (in λ) sequence of cardinals $\{\chi_i < \lambda : i < cf\lambda\}$ such that for all $i < cf\lambda$ $2^{\chi_i} = \chi_{i+1}$. If T is a first order theory of power $\le\lambda$, and Γ a non trivial smallness notion (see Def. 2.1 in (Sh)) then T has a model M of power λ^+ such that for every non algebraic formula φ we have $\varphi \in \Gamma \Leftrightarrow \varphi$ is realized by exactly λ elements of the model M.

Remarks. (1) The proof is done without expanding the language of T. (2) Assuming existence of a strongly compact cardinal there exists a model of ZFC + GCH + "the above theorem fails for $\lambda = \aleph_\omega$".

(Sh) Saharon Shelah, Models with second order properties I, Annals of Math. Logic 14(1978) 57-72.

(Received December 16, 1983)(Introduced by Saharon Shelah)

*84T-03-168 RAMI GROSSBERG and SAHARON SHELAH, Institute of Mathematics, The Hebrew University, Jerusalem, Israel. On the number of non isomorphic models of an infinitary theory which has the infinitary order property.

Let κ,λ infinite cardinals such that $\kappa\le\lambda$ (we have new information for the case $\kappa<\lambda$). Always T a theory in $L_{\kappa^+,\omega}$ of power $\le\kappa$, and $\varphi(\bar{x},\bar{y}) \in L_{\lambda^+,\omega}$. Now define $\mu^*(\lambda,T) = \mathrm{Min}\{\mu^*$: If T satisfy $(\forall\mu<\mu^*)(\exists M_\mu \models T)(\exists\{\bar{a}_i : i<\mu\} \subseteq |M_\mu|)(\forall i,j<\mu)(i<j \Leftrightarrow M_\mu \models \varphi(\bar{a}_i,\bar{a}_j))$ then $(\exists\varphi'(\bar{x},\bar{y}) \in L_{\kappa^+,\omega})(\forall\chi\ge\kappa)(\exists M_\chi \models T)(\exists\{\bar{a}_i : i<\chi\} \subseteq M_\chi)(i<j \Leftrightarrow M_\chi \models \varphi'(\bar{a}_i,\bar{a}_j))\}$.

The main concept is $\mu^*(\lambda,\kappa) = \mathrm{Sup}\{\mu^*(\lambda,T) : T \text{ a theory in } L_{\kappa^+,\omega} \text{ of power } \le\kappa\}$.

This is interesting because

Theorem 1. Let T and $\varphi(\bar{x},\bar{y})$ as above. If $(\forall\mu<\mu^*(\lambda,\kappa))(\exists M_\mu \models T)(\exists\{\bar{a}_i = i<\mu\} \subseteq M_\mu)(\forall i,j<\mu)(i<j \Leftrightarrow M_\mu \models \varphi(\bar{a}_i,\bar{a}_j))$ then for every $\chi>\kappa$ $I(\chi,T) = 2^\chi$.

Theorem 2. $\mu^*(\lambda,\aleph_0) = \beth_{\lambda^+}$.

Theorem 3. For every $\kappa\le\lambda$ we have $\mu^*(\lambda,\kappa) \le \beth_{(\lambda^\kappa)^+}$.

Theorem 4. For every $\kappa\le\lambda$, T as above, and any set of formulas $\Delta \subseteq L_{\lambda^+,\omega}$ such that $\Delta \supseteq L_{\kappa^+,\omega}$. If T is (Δ,μ)-unstable for μ satisfying $\mu^{\mu^*(\lambda,\kappa)} = \mu$ then T is Δ - unstable. Remarks (1) for κ or λ singular of cofinality $\aleph_0$ we have better bounds than in Theorem 3. (2) Similarly to Theorem 2 we have lower bounds also in other cases. (Received December 16, 1983)