

Mathematical Logic and Foundations (03, 04)

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An elementary set-theoretic proof of the inconsistency of ZFC. Part III.

Let BSC be a fragment of ZFC defined as in Abstract 779-03-11, ABSTRACTS AMS, Vol. 1 (1980) 404. Lemma (BSC). Let $F(Y)$ be a complete s -atomic field of subsets of Y and let A_F be the set of atoms of $F(Y)$. Let $\bigcup A_F$ be the union of A_F and let $\bigsqcup A_F$ be the supremum in $F(Y)$ of A_F . Then $\bigcup A_F = \bigsqcup A_F = Y$. Sketch of proof. The equality $\bigsqcup A_F = Y$ holds in $F(Y)$. Let A_P be the set of atoms of $P(Y) = \{z \mid z \subseteq Y\}$. There is a bijective homomorphism $s_P: P(Y) \rightarrow P(A_P)$ defined by $s_P(a) = \{t \in A_P \mid t \subseteq a\}$. There is a bijective homomorphism $s_F: F(Y) \rightarrow P(A_F)$ defined by $s_F(a) = \{t \in A_F \mid t \subseteq a\}$. Let $F(A_P) =_{df} s_P(F(Y))$ (image of $F(Y)$ under s_P). Let $s_b: F(Y) \rightarrow F(A_P)$ be defined by $\forall a \in F(Y) [s_b(a) = s_P(a)]$. Then s_b is a bijective homomorphism. Let $h: P(A_P) \rightarrow F(A_P)$ be defined by $h =_{df} s_F^{-1} \circ s_b$ (composite map of s_F^{-1} and s_b). Let $S(A_P) =_{df} \{u \in P(A_P) \mid u = \{t\} \wedge t \in A_P\}$. Let $e: S(A_P) \rightarrow F(A_P)$ be defined by $e =_{df} h \upharpoonright S(A_P)$ (restriction of h to $S(A_P)$). Then e is an inclusion map. Let $S(A_F) =_{df} \{u \in F(A_P) \mid u = \{t\} \wedge t \in A_P\}$. From the hypothesis $A_F \subseteq A_P$ ($F(Y)$ is s -atomic) follows $S(A_F) \subseteq S(A_P)$; therefore $\text{Range } e \subseteq S(A_P)$. Moreover, $\text{Range } e = S(A_P)$; hence e is onto. Thus e is an identity map. $S(A_F) = S(A_P) \Rightarrow A_F = A_P \Rightarrow \bigcup A_F = \bigcup A_P = Y$. Q.E.D. (Received June 6, 1980)

80T-E85 SAHARON SHELAH, Hebrew University of Jerusalem, Israel. Going to Canossa

The author had claimed the necessity of an inaccessible in Solovay proof of the consistency of ZF + every set of reals is measurable [has the property of Baire]. Some doubts disappear after the author proved a detailed proof of the measurability case. Unfortunately we have to withdraw the claim on the Baire property, but Th 1: If ZFC is consistent then so is ZFC + "every set of reals defined by a first order formula with real parameters has the Baire property. Subject B: Th. 2. If Namba forcing is semi proper then Chang conjecture holds. We formulate a condition forcing notions, s.t. suitable iteration of such forcings does not collapse \aleph_1 , and it is satisfied by Namba forcing and all semi-proper forcings: Subject C: Failure of MA's for \aleph_1 -complete forcings. Th.3: ($2^{\aleph_0} = \aleph_1$). There is an \aleph_1 -complete forcing notion P , such that a) $P = \bigcup_{i < \aleph_1} A_i$ and any countable subset of A_i has an upper bound b) there are dense $D_i \subseteq P(i < \aleph_2)$ such that there is no directed $G \subseteq P$, $G \cap D_i \neq \emptyset$ for $i < \aleph_2$. Th.4: ($2^{\aleph_0} = \aleph_1$). Let $S = \{\delta < \aleph_2 : cf \delta = \aleph_1\}$, there are $f_\delta: \delta \rightarrow \{0,1\}$ for $\delta \in S$ such that for any $f: \omega_2 \rightarrow \{0,1\}$ the following is a stationary subset of $\omega_2: \{\delta \in S \mid f \upharpoonright \delta = f_\delta\}$ is a stationary subset of δ . Subject D: Th.5: ($2^{\aleph_0} < 2^{\aleph_1}$) There is an abelian group G , of power \aleph_1 any endomorphism h of it is $h(x) = nx$ for some n which is \aleph_1 -free. (Received June 16, 1980)

80T-E86 THOMAS G. KUCERA, McGill University, Montreal, P.Q. H3A 2K6 Decomposition of injective modules over a Noetherian ring. Preliminary report.

As an application of a Basis Theorem for certain totally transcendental theories [these Abstracts,] we give a model-theoretic proof of some of the classic results of Matlis [Pac. J. Math 8 (1958) p.511] on injective modules. Let Λ be a Noetherian ring with 1, T_Λ^* the theory of linearly closed left Λ -modules of Eklof and Sabbagh [Ann. Math. Logic Vol. 2 No. 3. p. 251]. All models of T_Λ^* are injective and T_Λ^* is totally transcendental. We freely confuse a left ideal P with the 1-type P over \emptyset which says "Annihilator of x is P ". We define an ideal P to be "prime" if the type P is strongly regular. Any two such types which are non-orthogonal are in fact equal. The set \mathcal{P} of primes satisfies the hypothesis (*) of the Basis Theorem, so as a consequence we get a structure theorem for models of T_Λ^* , and a simple trick extends it to all injectives: Theorem (essentially Matlis): Let M be an injective left Λ -module. Then M has a unique decomposition $\bigoplus_{P \in \mathcal{P}} E(\Lambda/P)^{(\alpha_P)}$ as a direct sum of indecomposable injectives, and if $M \models T$ then $\alpha_P = \dim(P, M)$. The Basis Theorem was inspired by a study of the commutative case, where \mathcal{P} is in fact the set of (real) prime ideals. This result is obtained in a different manner than those given by Garavaglia [JSL Vol. 45 No. 1 p. 155]. This work will form part of the author's Ph.D. thesis. (Received June 30, 1980)