AN OVERVIEW OF THE PROOF IN "BOREL CONJECTURE AND DUAL BOREL CONJECTURE"

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Abstract. This note gives an informal overview of the proof in our paper "Borel Conjecture and Dual Borel Conjecture".

In this note, we give a rather informal overview (including two diagrams) of the proof given in our paper "Borel Conjecture and Dual Borel Conjecture" `http://arxiv.org/abs/1105.0823`, see also `https://shelah.logic.at/papers/969/view/` (let us call it the "main paper"). This overview was originally a section in the main paper (the section following the introduction), but the referee found it not so illuminating, so we removed it from the main paper.

Since we think the introduction may be helpful to some readers (as opposed to the referee, who found the diagrams "mystifying") we preserve it in this form.

The overview is supposed to complement the paper, and not to be read independently. In particular, we refer to the main paper for references.

The emphasis of this note is on giving the reader some vague understanding, at the expense of correctness of the claims (we point out some of the most blatant lies).

1. The general setup

We assume CH in the ground model. We use a σ-closed $\aleph_2$-cc preparatory forcing $\mathbb{R}$, which adds a generic "alternating iteration" (as defined below) $\bar{\mathbf{P}} = (\mathbf{P}_\alpha, \mathbf{Q}_\alpha)_{\alpha<\omega_2}$. Moreover, $\mathbb{R}$ forces that $\bar{\mathbf{P}}$ is ccc. The forcing notion to get BC+dBC is the composition $\mathbb{R} * \mathbf{P}_{\omega_2}$.

We say that $\bar{P}$ is an "alternating iteration" if $\bar{P} = (P_\alpha, Q_\alpha)_{\alpha<\omega_2}$ is a forcing iteration of length ω_2 satisfying the following:

- At every even step α, (P_α forces that) Q_α is an "ultralaver forcing" (described below).
- At every odd step α, (P_α forces that) Q_α is a "Janus forcing" (described below).
- However, instead of using either a Janus or an ultralaver forcing, we are at any step allowed just to "do nothing", i.e., set $Q_\alpha = \{\emptyset\}$.
- At a limit step δ, we take "partial countable support" limits. (This means more or less: P_δ is a subset of the countable support limit of $(P_\alpha)_{\alpha<\delta}$ and contains $\bigcup_{\alpha<\delta} P_\alpha$ and has some other natural properties.)

2. Ultralaver forcing

Let $\bar{D} = (D_s)_{s\in\omega^{<\omega}}$ be a system of ultrafilters. The "ultralaver forcing" $\mathbb{L}_{\bar{D}}$ consists of trees p with the following property: For every node $s \in p$ above the stem the set of immediate successors of s is in D_s. So this is a σ-centered variant of Laver forcing. Of course this forcing adds a naturally defined generic real, called ultralaver real.

We will basically need two properties of ultralaver forcing: The first one is preservation of positivity:

(2.1) $\qquad \mathbb{L}_{\bar{D}}$ preserves Lebesgue outer measure positivity of ground model sets.[1]

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The second one is killing of smz sets:

For every uncountable set X in the ground model, $\mathbb{L}_{\bar{D}}$ forces that X is non-smz.

Actually, we should formulate this claim in a stronger form. Let us first quote a result of Pawlikowski, which is essential for the part of our proof that shows BC:

Theorem 2.2. *$X \subseteq 2^\omega$ is smz iff $X + F$ is null for every closed null set F.*
Moreover, for every dense G_δ set H we can construct *(in an absolute way) a closed null set F such that for every $X \subseteq 2^\omega$ with $X + F$ null there is $t \in 2^\omega$ with $t + X \subseteq H$.*

So we can actually show the following:

(2.3) We can construct from the ultralaver real in an absolute way a (code for a) closed null set F such that $X + F$ is (outer Lebesgue measure) positive for every uncountable ground model set X.

It is an easy exercise to show that Theorem 2.2 implies the following fact.

Fact 2.4. Assume that $\bar{P} = (P_\alpha, Q_\alpha : \alpha < \omega_2)$ is an iteration with direct limit P_{ω_2} satisfying the following:

- For cofinally many $\alpha < \omega_2$, Q_α makes every old uncountable set non-smz.
- P_{ω_2} and even all quotients P_{ω_2}/P_α preserves Lebesgue outer measure positivity.
- P_{ω_2} preserves $\aleph_1$ and satisfies the $\aleph_2$-cc.

Then P_{ω_2} forces BC.

Remark 2.5. It is well-known that both Laver reals and random reals preserve positivity. As Laver forcing makes every old uncountable set non-smz, we conclude that a countable support iteration of length ω_2 of Laver reals, or alternatively, a countable support iteration alternating Laver with random reals, forces BC. The latter iteration also forces the failure of dBC, since the random reals increase the covering number of the null ideal, and every set smaller than this cardinal is sm.

3. A preparatory forcing for a single step

Let us first describe how to generically create a single forcing, e.g., an ultralaver forcing.

Let Q be a forcing, M a countable transitive model, $P \in M$ a subforcing of Q. We say that P is an M-complete subforcing of Q, if every maximal antichain $A \in M$ of P is also a maximal antichain in Q. In this case every Q-generic filter over V induces a P-generic filter over M.

Let M^x be a (countable) model, and $\bar{D}^x := (D^x_s)_{s\in\omega^{<\omega}}$ a system of ultrafilters in M^x. This defines the ultralaver forcing $Q^x = \mathbb{L}_{\bar{D}^x}$ in M^x. Given any system $\bar{D}$ of ultrafilters (in V) such that each D_s extends D^x_s, then we can show that Q^x is an M^x-complete subforcing of the ultralaver forcing $Q := \mathbb{L}_{\bar{D}}$ in V. We describe this by[2] "(M^x, Q^x) *canonically embeds into* Q".

So every Q-generic filter H over V induces a Q^x-generic filter over M^x which we call H^x. A trivial but crucial observation is the following: When we evaluate the ultralaver real for Q in $V[H]$ then we get the same real as when we evaluate it for Q^x in $M^x[H^x]$. Of course Q^x is not a complete subforcing of Q, just an M^x-complete one: While Q^x is just countable, therefore equivalent to Cohen forcing (from the point of view of V), the real added by the Q^x-generic H^x is an ultralaver real (over M^x as well as over V), and therefore does not add a Cohen real over V.

We can define a preparatory forcing $\mathbb{R}_{\mathbb{L}}$ (for a single ultralaver forcing) consisting of pairs $x = (M^x, Q^x)$ as above (M in some $H(\chi^*)$, say), and ordered as follows: y is stronger than x if $M^x \in M^y$ and (M^y thinks that) (M^x, Q^x) canonically embeds into Q^y. It is not hard to see that $\mathbb{R}_{\mathbb{L}}$ is σ-closed, and adds in the extension a generic ultralaver forcing $\mathbf{Q}$ such that each x in the generic filter embeds into $\mathbf{Q}$.

Let G be $\mathbb{R}_{\mathbb{L}}$-generic (over V). So in $V[G]$, we know that Q^x is an M^x-complete subforcing of $\mathbf{Q}$ for all $x \in G$. Let H be $\mathbf{Q}$-generic (over $V[G]$). Then H induces a Q^x-generic filter over M^x (which we call H^x)

[1] This is a lie, and moreover a stupid (i.e., useless) lie. It is a lie, since we only get something like: for one random over a specific model, we can find a system $\bar{D}$ such that $\mathbb{L}_{\bar{D}}$ preserves randomness. It is a useless lie, since preservation of positivity is not enough anyway: We need a stronger property that is preserved under proper countable support iterations.

[2] Note the linguistic asymmetry here: A symmetric and more verbose variant would say "(M^x, Q^x) canonically embeds into (V, Q)".

for all $x \in G$. Let us repeat the trivial observation: As above, each $M^x[H^x]$ will see the "real" ultralaver real (i.e., the one of $V[G][H]$).

Note that "canonical embedding" is a form of approximation: If x is in the generic filter, we do not know everything about $\mathbf{Q}$, but we know that $Q^x \subseteq \mathbf{Q}$ and that the maximal antichains of Q^x in M^x will be maximal antichains in $\mathbf{Q}$ as well.

We should at this stage mention another simple concept that will be used several times: Given (M^x, Q^x) and (in V) some ultralaver forcing Q such that (M^x, Q^x) embeds into Q, we can take some countable elementary submodel N of $H(\chi^*)$ containing (M^x, Q^x) and Q, and Mostowski-collapse (N, Q) to $y = (M^y, Q^y)$. Then y is in $\mathbb{R}_{\mathbb{L}}$ and stronger than x.

4. Janus forcing

With "Janus forcings" we denote a family of forcing notions (as in the case of "ultralaver forcings"). Every Janus forcing $\mathbb{J}$ is a subset of $H(\aleph_1)$ and has a countable "core" ∇ (which is the same for every Janus forcing) and some additional "stuffing".[3] The forcing ∇ will add a generic real ("Janus real") coding a null set Z_∇. The forcing ∇ will not be a complete subforcing of $\mathbb{J}$, but we will require that all maximal antichains involved in the name Z_∇ are also maximal in $\mathbb{J}$, so that $\mathbb{J}$ also adds a generic null set Z_∇.

The crucial combinatorial content of Janus forcings heavily relies on previous work by Bartoszyński and Shelah.

Analogously to the case of ultralaver forcing, let $\mathbb{R}_{\mathbb{J}}$ consist of pairs $x = (M^x, Q^x)$ such that M^x is a countable model and Q^x a Janus forcing in M^x. Given $x \in \mathbb{R}_{\mathbb{J}}$ and a Janus forcing Q in V, we say that x canonically embeds into Q if Q^x is an M^x-complete subforcing of Q, and we set $y \leq x$ in $\mathbb{R}_{\mathbb{J}}$ if M^y thinks that x canonically embeds into Q^y. Again $\mathbb{R}_{\mathbb{J}}$ is a σ-closed forcing and adds a generic object $\mathbf{Q}$ that is (forced to be) a Janus forcing; and for every x in the $\mathbb{R}_{\mathbb{J}}$-generic filter, x canonically embeds into $\mathbf{Q}$.

As in the case of ultralaver forcing, the Janus real Z_∇ is absolute.

As opposed to the case of ultralaver forcing, every Janus forcing Q^x in any model M^x is itself a Janus forcing in V; so (taking the collapse of an elementary submodel as above) we trivially get:

(4.1) For every $x \in \mathbb{R}_{\mathbb{J}}$ there is a $y \leq x$ such that in M^y, Q^x is a countable Janus forcing (so in particular equivalent to Cohen forcing).

A crucial property of Janus forcing is that we can make it into random forcing as well:

(4.2) For every $x \in \mathbb{R}_{\mathbb{J}}$ there is a $y \leq x$ such that in M^y, Q^y is forcing equivalent to random forcing.

So here we see an important property of the preparatory forcing $\mathbb{R}_{\mathbb{J}}$, which might seem a bit paradoxical at first: "Densely", $\mathbf{Q}$ seems to be Cohen as well as random forcing. This two-faced behavior gives Janus forcing its name; one could also describe this behavior as "faking" (faking to be Cohen and faking to be random).

Janus forcing is the forcing notion that replaces the Cohen real in the dBC part of the proof. The crucial point is:

> A countable Janus forcing makes every uncountable ground model set of reals non-sm.

Well, that is actually not much of a point at all: As Carlson has shown, this is achieved by a Cohen real, and obviously a countable Janus forcing is equivalent to a Cohen real. And Carlson even showed: When adding a Cohen real, this uncountable ground model set remains non-sm even after forcing with another forcing notion, provided this forcing notion has precaliber $\aleph_1$.

So what we actually claim for Janus forcing is the more "explicit" version of our trivial (after Carlson) claim above. First, let us recall the (trivially modifies) definition of strongly meager (sm): notation:

(4.3) X is *not* sm, iff there a null set Z (called "witness") such that $X + Z = 2^\omega$.

[3] Actually, the definition of Janus forcing additionally depends on a real parameter. In our application, we will use ultralaver forcings as even stages α, and use a Janus forcing defined from the ultralaver real in the stage $\alpha + 1$. The following claims about Janus forcings only hold for this situation; in particular the ground-model sets mentioned have to live in the model before the ultralaver forcing.

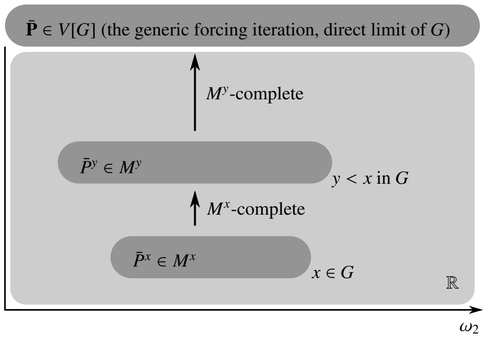

FIGURE 1. G is the generic filter for the preparatory forcing notion $\mathbb{R}$, which adds the generic iteration $\bar{\mathbf{P}}$. The forcing that gives BC+dBC is $\mathbb{R} * \mathbf{P}_{\omega_2}$. (Of course, in contrast to the impression given by the diagram, the set $M^x \cap \omega_2$, which is the "domain" of $\bar{P}^x$, is not an interval.)

So what we really claim is the following:

(4.4) The canonical null set Z_∇ added by a countable Janus forcing has the property that $X + Z_\nabla = 2^\omega$ for all uncountable ground model sets X, and moreover $X + Z_\nabla = 2^\omega$ is preserved by every subsequent σ-centered forcing.

Of course Z_∇ is interpreted as a code for a null set, not a concrete subset of the reals (otherwise $X + Z_\nabla = 2^\omega$ could not hold when we add new reals).

Let us again note that it is important that we can construct the null set Z_∇ (rather: the code) in an absolute way from the Janus real and get (4.4).

5. The preparatory forcing for the iteration

The preparatory forcing $\mathbb{R}$ that we will use will be similar to $\mathbb{R}_\mathbb{L}$ or to $\mathbb{R}_\mathbb{J}$, but instead of "approximating" a single generic ultralaver or Janus forcing, we approximate the alternating iteration $\bar{\mathbf{P}}$ mentioned in Section 1. So our preparatory forcing $\mathbb{R}$ consists of pairs $x = (M^x, \bar{P}^x)$, where M^x is a countable model[4] (and subset of some fixed $H(\chi^*)$) and $\bar{P}^x$ is in M^x an alternating iteration.

Assume that $x \in \mathbb{R}$ and that $\bar{P}$ is (in V) an alternating iteration. As opposed to the case of a single ultralaver forcing, we now cannot formally assume that $\bar{P}^x$ is a subset of $\bar{P}$, but there is a natural construction that tries to give an M^x-complete embedding of $\bar{P}^x$ into $\bar{P}$. If this construction works, we say that "*x is canonically embeddable into $\bar{P}$*", and in that case we can treat $\bar{P}^x$ as subset of $\bar{P}$. So if x is canonically embeddable into $\bar{P}$, and H is a P_{ω_2}-generic filter over V (which of course induces P_α-generic filters H_α for all $\alpha \leq \omega_2$), we can get a canonical P^x_α-generic filter over M^x for every $\alpha \in \omega_2 \cap M^x$ (which we call H^x_α).

We define the order in $\mathbb{R}$ as above: For $x, y \in \mathbb{R}$, we define y to be stronger than x, if $M^x \in M^y$ and M^y thinks that x canonically embeds into $\bar{P}^y$.

Note that while M^x thinks that $\bar{P}^x$ is an iteration of length ω_2, in V (or in M^y for $y \leq x$) the "real domain" of $\bar{P}^x$ is just countable (since it is a subset of M^x).

As promised, one can show that $\mathbb{R}$ is σ-closed and adds a generic alternating iteration $\bar{\mathbf{P}}$, and that $\bar{\mathbf{P}}$ is ccc. The final limit $\mathbf{P}_{\omega_2}$ is the direct limit of the $\mathbf{P}_\alpha$ (and thus does not add any new reals in the last stage). The intermediate stages satisfy CH, while $\mathbf{P}_{\omega_2}$ forces $2^{\aleph_0} = \aleph_2$. As might be expected by now, each x in the $\mathbb{R}$-generic filter G canonically embeds into $\bar{\mathbf{P}}$. We will call the $\mathbb{R}$-generic filter G. (The situation is illustrated in Figure 1.) So if H is $\mathbf{P}_{\omega_2}$-generic over $V[G]$, then we get canonical $P^x_{\omega_2}$-generic filters H^x for all $x \in G$; and the "real" ultralaver (and Janus) reals calculated in $V[G][H]$ are the same as the ones "locally" calculated in $M^x[H^x]$.

Given any $x \in \mathbb{R}$ we can construct (in V) an alternating iteration $\bar{P}$ such that x embeds into $\bar{P}$ and such that $\bar{P}$ has either of the following two properties:

[4] Since we are interested in iterations of length ω_2, we cannot use transitive models, that can only see ordinals $< \omega_1$. Instead, we use ord-transitive models.

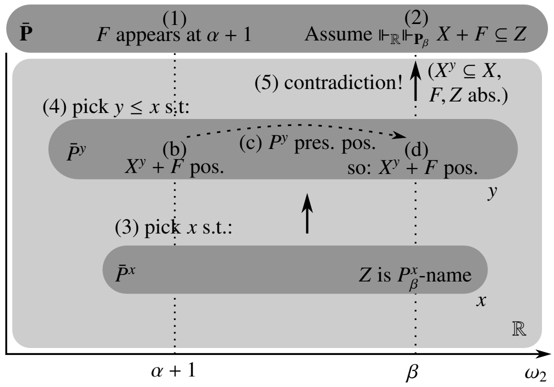

FIGURE 2. The proof of BC.

- All Janus forcings are countable; at all stages α that are not in M^x we "do nothing"; and all limits P_δ are "almost finite support over x". (I.e., basically the limit is finite support, but we more or less add the countably many elements of P^x_δ.)
- All Janus forcings are equivalent to random forcing, and all limits P_δ are "almost countable support over x" (basically we take all conditions in the countable support limit that are x-generic).

The point is that these iterations behave more or less like finite (or countable) support iterations; but we can still embed x into them. For example, M^x could think that $\bar{P}^x$ is a countable support iteration, but we may still choose $\bar{P}$ to be an almost finite support iteration.

"Behave more or less in the same way" implies in particular in the first case that any P_α is σ-centered: We iterate only countably many forcings, since we do nothing outside M^x; the single forcings are σ-centered (in the ultralaver case) and even countable in the Janus case, and the (almost) finite support limits preserve σ-centeredness.

In the second case, we get preservation of positivity (with respect to outer Lebesgue measure): ultralaver as well as random forcings preserve positivity, and preservation is preserved by (almost) countable support (proper) iterations.[5]

As above, we put the iteration $\bar{P}$ into a countable elementary submodel; collapse it, and thus get:

(5.1) For all $x\in\mathbb{R}$ there is a $y\le x$ such that (M^y thinks that) $P^y_{\omega_2}$ is σ-centered and all Janus forcings are countable.

(5.2) For all $x\in\mathbb{R}$ there is a $y\le x$ such that (M^y thinks that) $P^y_{\omega_2}$ preserves Lebesgue outer measure positivity.

Let us again note that densely often we use finite support, but we also use countable support densely often.

6. WHY BC HOLDS

We want to show that BC is forced by $\mathbb{R}*\mathbf{P}_{\omega_2}$. Let X be the name of a set of reals of size $\aleph_1$. Since $\mathbf{P}_{\omega_2}$ has length ω_2, we can assume[6] that X is in the ground model V. We want to show BC, so we have to show that X is not smz. The following is illustrated by Figure 2.

(1) Fix any ultralaver position α. (Well, we fix α large enough to justify our assumption that $X\in V$.) We know that the ultralaver real that is added by $\mathbf{Q}_\alpha$ (i.e., appears at stage $\alpha+1$) defines in an absolute way a (code for a) closed null set F.

(2) According to Theorem 2.2, it is enough to show that $X+F$ is non-null in the extension by $\mathbb{R}*\mathbf{P}_{\omega_2}$ (where the Borel code F is evaluated in the extension). So assume towards a contradiction that

[5] Of course, this is not true, rather we need an iterable property such as preservation of random reals over models, etc. We do not get this stronger property universally, we can just preserve a specific random; so claim (5.2) is a lie, too.

[6] Well, we can't. But we can do something similar, as will be explained in the final section of the main paper.

$X + F$ is forced to be a subset of a null set (or rather, a Borel code) Z; this already has to happen[7] at some stage $\beta < \omega_2$. In other words: We assume (towards a contradiction)

$$\Vdash_{\mathbb{R}} \Vdash_{\mathbf{P}_\beta} X + F \subseteq Z.$$

(3) Since $\mathbf{P}_\beta$ is (forced to be) ccc, we can find a very "absolute" (countable) name for Z; and we can find an $x \in \mathbb{R}$ that already calculates Z correctly.[8]
(4) Now we construct (in V) a $y \leq x$ in $\mathbb{R}$ (with $\bar{P}^y_\alpha$ proper) that satisfies (5.2), and moreover such that $X^y := X \cap M^y \in M^y$ is uncountable in M^y (we get this for free if M^y is the collapse of an elementary submodel N with $X \in N$) In particular, (M^y thinks that)
  (a) P^y_α is proper, thus preserves $\aleph_1$, thus forces that X^y is uncountable.
  (b) Therefore Q^y_α forces that $X^y + F$ is positive (according to (2.3)).
  (c) $P^y_{\omega_2}$ preserves positivity.
  (d) Therefore P^y_β forces that $X^y + F$ is positive[9] (and in particular not a subset of Z).
(5) This leads to the obvious contradiction: Let G be $\mathbb{R}$-generic over V and contain y, and let H_β be $\mathbf{P}_\beta$-generic over $V[G]$. Then H^y_β is P^y_β-generic over M^y, and therefore $M^y[H^y_\beta]$ thinks that some $x + f \in X^y + F$ is not in Z. But $X^y = X \cap M^y \subseteq X$, and the codes for F and for Z are absolute (for F since it is constructed in a canonical way from the ultralaver real, for Z because we took care of it in step (3)).

7. Why dBC holds

The proof of dBC is similar, using σ-centeredness instead of positivity preserving, and a countable Janus forcing instead of ultralaver forcing.

Let X be the name of a set of reals of size $\aleph_1$. Again, without loss of generality[10] $X \in V$. We want to show dBC, so we have to show that X is not sm.

(1) Fix any Janus position α (large enough to justify our assumption that $X \in V$). We know that the Janus real that is added by $\mathbf{Q}_\alpha$ (i.e., appears at stage $\alpha + 1$) defines in an absolute way a (code for a) null set Z_∇.
(2) According to (4.3), it is enough to show that $Z_\nabla + X = 2^\omega$ in the extension by $\mathbb{R} * \mathbf{P}_{\omega_2}$. So assume towards a contradiction that $Z_\nabla + X \neq 2^\omega$. This already happens at some stage $\beta < \omega$, i.e., we assume

$$\Vdash_{\mathbb{R}} \Vdash_{\mathbf{P}_\beta} r \notin Z_\nabla + X.$$

(3) Again, find x such that r is an "absolute" P^x_β-name.
(4) Now we construct (in V) a $y \leq x$ in $\mathbb{R}$ that satisfies (5.1), and such that $X^y := X \cap M^y \in M^y$ is uncountable in M^y. In particular, (M^y thinks that)
  (a) P^y_α is proper, thus preserves $\aleph_1$, and so forces that X^y is uncountable.
  (b) Q^y_α is (forced to be) a countable Janus forcing notion.
  (c) $P^y_{\omega_2}$ is σ-centered.
  (d) Therefore (4.4) implies that P^y_β forces that $Z_\nabla + X^y = 2^\omega$, in particular that $r \in Z_\nabla + X^y$.
(5) As before, this leads to a contradiction.

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[7]Each real (and in particular the Borel code Z) in the $\mathbf{P}_{\omega_2}$-extension already has to appear at some stage $\beta < \omega_2$; and the statement "$X + F \subseteq Z$" is absolute.

[8]More formally: We find an $x \in \mathbb{R}$ and a P^x_β-name Z^x in M^x such that x forces (in $\mathbb{R}$) that $\mathbf{P}_\beta$ forces that Z (evaluated by the $\mathbf{P}_\beta$-generic) is the same as Z^x (evaluated by the induced P^x_β-generic).

[9]Here we even get positivity of $X^y + F$ where F is evaluated in the intermediate extension of stage $\alpha + 1$. However, we get the contradiction even if we just assume that $X^y + F$ is positive where F is evaluated in the P^y_β-extension.

[10]And again, this is a lie.

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