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THE AUTOMORPHISM GROUP OF HALL'S UNIVERSAL GROUP

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Abstract. We study the automorphism group of Hall's universal locally finite group H. We show that in $Aut(H)$ every subgroup of index $< 2^{\aleph_0}$ lies between the pointwise and the setwise stabilizer of a unique finite subgroup A of H, and use this to prove that $Aut(H)$ is complete. We further show that $Inn(H)$ is the largest locally finite normal subgroup of $Aut(H)$. Finally, we observe that from the work of the second author it follows that for every countable locally finite G there exists $G \cong G' \leqslant H$ such that every $f \in Aut(G')$ extends to an $\hat{f} \in Aut(H)$ in such a way that $f \mapsto \hat{f}$ embeds $Aut(G')$ into $Aut(H)$. In particular, we solve the three open questions of Hickin on $Aut(H)$ from his 1978 work, and give a partial answer to Question VI.5 of Kegel and Wehrfritz from their 1973 work.

1. Introduction

In [2] Hall constructs a group H with the following properties:

(A) H is countable;
(B) H is locally finite;
(C) H embeds every finite group;
(D) any two isomorphic finite subgroups of H are conjugate in H.

The group H is unique modulo isomorphism and it is known as *Hall's universal locally finite group* (or simply as Hall's universal group). In model-theoretic terminology H is a homogeneous structure, i.e. a structure M such that every isomorphism between finitely generated substructures of M extends to an automorphism of M. Groups of automorphisms of such structures have received extensive attention in the literature (see e.g. [9], [10], [5] and [17]). Despite this, not much is known on $Aut(H)$. In this paper we make progress in this direction proving the following theorems:

Theorem 1. *Every subgroup of $Aut(H)$ of index less than $2^{\aleph_0}$ lies between the pointwise and the setwise stabilizer of a unique finite subgroup A of H.*

Theorem 2. *$Aut(H)$ is complete (i.e. $Aut(H)$ has no center and no outer automorphisms).*

Theorem 3. *$Inn(H)$ is the locally finite radical of $Aut(H)$ (i.e. it is the largest locally finite normal subgroup of $Aut(H)$).*

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Theorem 4. *For every countable locally finite G there exists $G \cong G' \leqslant H$ such that every $f \in Aut(G')$ extends to an $\hat{f} \in Aut(H)$ in such a way that $f \mapsto \hat{f}$ embeds $Aut(G')$ into $Aut(H)$.*

In particular, we solve the three open questions of Hickin on $Aut(H)$ from [4] (see pg. 227), and give a partial answer to Question VI.5 of Kegel and Wehrfritz from [7].

After the writing of this paper, thanks to the referee, we discovered that our Theorem 2 is implied by a known result, i.e. that non-abelian simple groups have complete automorphism groups (see e.g. [1], where this is attributed to Burnside). In fact, by [7, Theorem 6.1], H is simple and so by the above we immediately get that $Aut(H)$ is complete. Nonetheless, we believe that our proof is enlightening and that the underlying ideas could be used to establish the completeness of the automorphism groups of other combinatorial and algebraic structures with the so-called strong small index property (cf. Definition 6).

2. The strong small index property for $Aut(H)$

In this section we prove Theorems 6 and 4.

Proof of Theorem 4. This is implicitly proved in [16, Claim 3.13(1) and 3.15]. □

As an immediate consequence of Theorem 4, we answer positively to the first two open questions of Hickin on $Aut(H)$ from [4] (see pg. 227).

Corollary 5. (1) *$Aut(H)$ embeds the symmetric group $Sym(\omega)$.*
(2) *There is an infinite set $S \subseteq H$ such that every permutation of S can be lifted to an automorphism of H.*

Proof. Let G be the countably infinite dimensional vector space over the field of order 2, and G' and $F : f \mapsto \hat{f}$ as in Theorem 4. Let S be a basis for G' and $A(S)$ the subgroup of $Aut(G')$ of automorphisms induced by permutations of S. Then F witnesses that every permutation of S extends to an automorphism of H, and $F \restriction A(S)$ embeds $A(S) \cong Sym(\omega)$ into $Aut(H)$. □

Definition 6. Let M be a countable structure and $G = Aut(M)$. We say that M (or G) has the *small index property* if every subgroup of $Aut(M)$ of index less than $2^{\aleph_0}$ contains the pointwise stabilizer of a finite set $A \subseteq M$.

Proof of Theorem 6. We first show that H has the small index property. By [6, Theorem 6.9] it suffices to show that $Aut(H)$ admits ample generics. To see this, by Sections 6.1 and 6.2 of [6] it suffices to show that the class of finite groups has the extension property for partial automorphisms and the amalgamation property for automorphisms. The first follows directly from the corollary on pg. 538 of [11], and the second is proved in [16, Claim 2.8]. The theorem now follows from the small index property, the main result of [15] and [16, Claim 2.8]. □

3. Completeness of $Aut(H)$

In this section we prove Theorem 2. To prove this we need the technology introduced in [14], which we briefly review below.

Let H be Hall's group and $G = Aut(H)$. We denote by $\mathbf{A}(H) = \{K \leqslant_{fin} H\}$ (where $K \leqslant_{fin} H$ means that $K \leqslant H$ and K is finite), and by $\mathbf{EA}(H) = \{(K, L) : K \in \mathbf{A}(H)$ and $L \leqslant Aut(K)\}$.

Let $(K, L) \in \mathbf{EA}(H)$, and we define:

$$G_{(K,L)} = \{h \in Aut(H) : h \restriction K \in L\}.$$

Notice that if $L = \{id_K\}$, then $G_{(K,L)} = G_{(K)}$, i.e. it equals the pointwise stabilizer of K, and that if $L = Aut(K)$, then $G_{(K,L)} = G_{\{K\}}$, i.e. it equals the setwise stabilizer of K. We then let:

$$\mathcal{PS}(H) = \{G_{(K)} : K \in \mathbf{A}(H)\} \text{ and } \mathcal{SS}(H) = \{G_{(K,L)} : (K, L) \in \mathbf{EA}(H)\}.$$

Let $\mathbf{L}(H)$ be a set of finite groups such that for every $K \in \mathbf{A}(H)$ there is a unique $L \in \mathbf{L}(H)$ such that $L \cong Aut(K)$.

Definition 7. We define the structure $ExAut(H)$, the *expanded group of automorphisms of* H, as follows:

(1) $ExAut(H)$ is a two-sorted structure;
(2) the first sort has set of elements $Aut(H) = G$;
(3) the second sort has set of elements $\mathbf{EA}(H)$;
(4) we identify $\{(K, id_K) : K \in \mathbf{A}(H)\}$ with $\mathbf{A}(H)$;
(5) the relations are:
 (a) $P_{\mathbf{A}(H)} = \{K \in \mathbf{A}(H)\}$ (recalling the above identification);
 (b) for $L \in \mathbf{L}(H)$, $P_{L(H)} = \{K \in \mathbf{A}(H) : Aut(K) \cong L\}$;
 (c) $\leqslant_{\mathbf{EA}(H)} = \{((K_1, L_1), (K_2, L_2)) : (K_i, L_i) \in \mathbf{EA}(H)\ (i = 1, 2),\ K_1 \leqslant K_2$ and $L_2 \restriction K_1 \leqslant L_1\}$;
 (d) $\leqslant_{\mathbf{A}(H)} = \{(K_1, K_2) : K_i \in \mathbf{A}(H)\ (i = 1, 2)$ and $K_1 \leqslant K_2\}$;
 (e) $P^{min}_{\mathbf{A}(H)} = \{K \in \mathbf{A}(H) : \{e\} \neq K \in \mathbf{A}(H)$ is minimal in $(\mathbf{A}(H), \subseteq)\}$;
(6) the operations are:
 (f) composition on $Aut(H)$;
 (g) for $f \in Aut(H)$ and $K \in \mathbf{A}(H)$, $Op(f, K) = f(K)$;
 (h) for $f \in Aut(H)$ and $(K_1, L_1) \in \mathbf{EA}(H)$, $Op(f, (K_1, L_1)) = (K_2, L_2)$ iff $f(K_1) = K_2$ and $L_2 = \{f \restriction K_1 \pi f^{-1} \restriction K_2 : \pi \in L_1\}$.

We say that a set of subsets of a structure N is second-order definable if it is preserved by automorphisms of N. We say that a structure M is second-order definable in a structure N if there is an injective map $\mathbf{j}$ mapping $\emptyset$-definable subsets of M to second-order definable set of subsets N.

Theorem 8. (1) *The map* $\mathbf{j}_H = \mathbf{j} : (h, (K, L)) \mapsto (h, G_{(K,L)})$ *witnesses second-order definability of* $ExAut(H)$ *in* $Aut(H)$.
(2) *Every* $f \in Aut(G)$ *has an extension* $\hat{f} \in Aut(ExAut(H))$.

Proof. This is because of Theorem 6 and [14, Theorem 12]. □

Before proving Theorem 2 we need a crucial lemma.

Lemma 9. *Let* $K_1, K_2 \leqslant_{fin} H$ *realizing the same quantifier-free type in* $ExAut(H)$.

(1) *If* K_1 *has prime order, then* $K_1 \cong K_2$.
(2) *If* K_1 *is abelian, then so is* K_2.
(3) *If* K_1 *is cyclic, then so is* K_2.
(4) *If* K_1 *is cyclic of order* n, *then* $K_1 \cong K_2$.
(5) K_1 *and* K_2 *have the same order.*
(6) *If* K_1 *and* K_2 *are with no center and* K_1 *is complete, then* $K_1 \cong K_2$.
(7) *If* K_1 *has no characteristic subgroup, then so does* K_2.
(8) *If* K_1 *is the alternating group on* $n > 6$, *then* $K_1 \cong K_2$.

Proof. (1) As groups of prime order are the only groups without non-trivial subgroups, and if $p \neq q$ are prime, then $Aut(C_p) \ncong Aut(C_q)$.
(2) A finite group K is abelian if and only if there is cyclic $L_1 \leqslant Aut(K)$ and $K^* \geqslant K$ such that for no $L_2 \leqslant Aut(K^*)$ we have $\{f \restriction K : f \in L_2\} = L_1$.
(3) A finite group K is cyclic if and only if it is abelian and there is a finite number of primes P such that for every $p \in P$ there is a unique $K_1 \leqslant K$ of order p.
(4) By (4) it suffice to define $|K|$ for cyclic K. Let $|K| = \prod_{i<k} p_i^{n_i}$, for $(p_i)_{i<k}$ a sequence of primes with no repetitions and $n_i \geqslant 1$. Notice now the following:

(i) We can define $\{p_i : i < k\}$.
(ii) For every $i < k$, we can define $\{K' \leqslant K : p \mid |K'|\}$.
(iii) For every $i < k$, $|\{K' \leqslant K : p_i \mid |K'|\}| = n_i$.

(5) If K_1 is a finite group, then $|K_1| = 1 + \sum\{m_K : K \leqslant K_1 \text{ cyclic}\}$, where, if $|K| = n$, $m_K = |\{a \in \{1, ..., n-1\} : (a, n) = 1\}$. Thus, by (4) we are done.
(6) By the choice of $ExAut(H)$ we have $Aut(K_1) \cong Aut(K_2)$. By (5) we have $|K_1| = |K_2|$. Hence, since K_1 is complete, $|Aut(K_2)| = |Aut(K_1)| = |K_1| = |K_2|$. Since K_2 is centerless we have $K_2 \cong Aut(K_2)$, and so we are done.
(7) By the choice of $ExAut(H)$ (cf. the operation Op).
(8) Since K_1 is the alternating group on $n > 6$, K_1 has no characteristic subgroup. Thus, by (7), also K_2 does not have a characteristic subgroup. Furthermore, by the proof of (2) and the fact that K_1 is not abelian, we have that K_2 is not abelian either. Hence, the center of K_2 is properly contained in K_2, and so it is the identity, since K_2 has no characteristic subgroup. Let $\pi_0 : Alt(n) \cong K_1$, π_1 be an embedding of $Sym(n)$ into H extending π_0 and $K_1^+ = ran(\pi_1)$. Let $K_2^+ \in \mathbf{A}(H)$ be such that $K_2 \leqslant K_2^+$, and (K_1, K_1^+) and (K_2, K_2^+) realize the same type. In particular, $|K_1^+| = |K_2^+|$ and $[K_2^+ : K_2] = 2$. We claim that K_2^+ is centerless. In fact, suppose otherwise, and let $K_1^+ \leqslant K_1^{++} \leqslant H$ be such that $K_1^{++} = K_1^+ \oplus K_0$ with $|K_0| = 2$. Then $Aut(K_1^{++}) \cong Aut(K_1^+)$. But K_2^+ does not have such an extension, which contradicts the choice of K_2^+. Hence, K_2^+ is centerless, and so, by (6) and the fact that $n > 6$, there exists $\pi : K_1^+ \cong K_2^+$. Now, K_1^+ has a unique subgroup of index 2, and so the same holds for K_2^+. Hence, π has to map K_1 onto K_2. □

We now prove Theorem 2.

Proof of Theorem 2. Let $f \in Aut(G)$ and $\hat{f}$ be the corresponding extension of f to $Aut(ExAut(H))$. Now, $\hat{f}$ maps $P^{min}_{\mathbf{A}(H)} \cap P_{e(H)}$ onto itself, where e denotes the trivial group. Clearly,

$$P^{min}_{\mathbf{A}(H)} \cap P_{e(H)} = \{K \leqslant H : |K| = 2\},$$

since the groups of order 2 are the only rigid groups without non-trivial subgroups. Thus, $\hat{f}$ induces a permutation g_1 of $\mathcal{X}_2(H) = \{x \in H : x \text{ has order } 2\}$.

Claim 1. The map g_1 can be extended to a $g_2 \in G$.

Proof of Claim 1. As $\mathcal{X}_2(H)$ generates H, it suffices to prove that if $x_1, ..., x_n \in \mathcal{X}_2(H)$ for $n > 3$, then there are $K_1, K_2 \leqslant_{fin} H$ such that:

(i) $x_1, ..., x_n \in K_1$;
(ii) $g_1(x_1), ..., g_1(x_n) \in K_2$;
(iii) there is an isomorphism h from K_1 onto K_2 such that $\bigwedge_{0<i\leqslant n} h(x_i) = g_1(x_i)$.

Let K_0 be the subgroup of H generated by $\{x_1, ..., x_n\}$ and $n_* = 2|K_0|$. Then we can find $K_1 \geqslant K_0$ which is isomorphic to the alternating group on n_*. Thus, by Lemma 9, letting $K_2 = \hat{f}(K_1)$ we are done. $\square$

Let $f_1 \in Aut(G)$ be such that $h \mapsto g_2 h g_2^{-1}$. We claim that $f_2 := f_1^{-1} f = id_G$. Towards contradiction, suppose there exists $h_1 \in G$ such that $h_2 := f_2(h_1) \neq h_1$. Since $\mathcal{X}_2(H)$ generates H, we can find $x_0 \in \mathcal{X}_2(H)$ such that:

$$x_1 := h_1(x_0) \neq h_2(x_0) := x_2.$$

Thus,

$$\begin{aligned} h_1 G_{\{e,x_0\}} h_1^{-1} = G_{\{e,x_1\}} \quad &\Rightarrow \quad f_2(h_1) f_2(G_{\{e,x_0\}}) f_2(h_1^{-1}) = f_2(G_{\{e,x_1\}}) \\ &\Rightarrow \quad h_2 G_{\{e,x_0\}} h_2^{-1} = G_{\{e,x_1\}} \\ &\Rightarrow \quad h_2(x_0) = x_1, \end{aligned}$$

which is absurd. Hence, $f_2 = id_G$, and so $f = f_1 \in Inn(G)$, as wanted. $\square$

4. $Inn(H)$ is the locally finite radical of $Aut(H)$

In this section we prove Theorem 3, which solves the third question of Hickin[1] on $Aut(H)$ from [4] (see pg. 227). We first need some facts and a proposition.

Fact 10 ([8])**.** Let $K \leqslant_{fin} H$. Then $\mathbf{C}_H(K)$ is isomorphic to an extension of $Z(K)$ by H (i.e. $\mathbf{C}_H(K)/Z(K) \cong H$).

Fact 11 ([3][Lemma 2.3])**.** Let $A \lneq B$ and C be finitely generated subgroups of an algebraically closed group G and $f \in Aut(G) - Inn(G)$. Then there exists in G an isomorphic copy B' of B over A (i.e. $a' = a$ for every $a \in A$) such that $f(B') \nsubseteq \langle B', C \rangle_G$.

Proposition 12. *Let $f \in Aut(H) - Inn(H)$ be of finite order $n < \omega$, and $K \leqslant_{fin} H$. Then there are commuting $a \neq b \in \mathbf{C}_H(K) - K$ of order 2 such that $f(a) = b$.*

Proof. By Fact 10 we can find $a \in \mathbf{C}_H(K) - K$ of order 2, since H is generated by elements of order 2. Similarly, letting $A = \langle f^{\pm i}(a), f^{\pm i}(K) : i < n \rangle_H$, we can find $b'' \in \mathbf{C}_H(\langle f^{-1}(K), f^{-1}(a) \rangle_H) - A$ also of order 2. Let now A be as above, $B = \langle A, b'' \rangle_H$ and $C = \{e\}$. Then, by Fact 11, there exists $h : B' \cong_A B$ such that $f(B') \nsubseteq B'$. Notice that $f(A) \subseteq A$, since f is of finite order n. Thus, letting $b' = h(b'')$ and $f(b') = b$, we must have that $b \notin B'$, and so in particular $b \neq a$ and $b \notin K$, since $A \subseteq B'$. Furthermore, by the choice of b'' and that of (A, B) we have:

$$\begin{aligned} b'' \in \mathbf{C}_H(\langle f^{-1}(K), f^{-1}(a) \rangle_H) \quad &\Rightarrow \quad b' \in \mathbf{C}_H(\langle f^{-1}(K), f^{-1}(a) \rangle_H) \\ &\Rightarrow \quad b \in \mathbf{C}_H(\langle K, a \rangle_H), \end{aligned}$$

since $B' \cong_A B$ and $\langle f^{-1}(K), f^{-1}(a) \rangle_H \leqslant A$. $\square$

Finally, we prove Theorem 3. For $c \in H$ we denote conjugation by c by $\square_c$.

Proof of Theorem 3. Let $N \trianglelefteq H$ properly containing $Inn(H)$. We want to show that N is not locally finite. Let then $f \in N - Inn(H)$. If f is of infinite order we are done. So suppose f has finite order. We construct $g \in Aut(H)$ such that

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[1] According to Hickin this question was posed by J. E. Roseblade; see [4] pg. 227.

$g^{-1}f^{-1}gf$ has infinite order. Let $\{d_i : i < \omega\} = H$. By induction on $i < \omega$, we define $K_i \leqslant_{fin} H$, $c_i \in H$, $(a_{2i-1}, a_{2i}), (b_{2i-1}, b_{2i}) \in H^2$ such that for $i < k < \omega$:

(i) $f(a_i) = b_i$;
(ii) $a_i \neq a_k$ and $b_i \neq b_k$ and $\{a_j : j < i\} \cap \{b_j : j < i\} = \emptyset$;
(iii) $\langle a_j, b_j : j < i\rangle_H = \langle a_j : j < i\rangle_H \oplus \langle b_j : j < i\rangle_H \cong (\mathbb{Z}_2)^i \oplus (\mathbb{Z}_2)^i$;
(iv) $K_i \leqslant K_k$;
(v) $(d_0, ..., d_{i-1}) \in K_i^{<\omega}$;
(vi) $(a_0, ..., a_{2i-1}), (b_0, ..., b_{2i-1}), (c_0, ..., c_i) \subseteq K_i^{<\omega}$;
(vii) $a_{2i}, b_{2i} \in \mathbf{C}_H(K_i)$;
(viii) $c_i \in \mathbf{C}_H(K_{i-1})$ and $\Box_{c_i}$ maps $b_{2(i-1)}$ to b_{2i-1} and a_{2_i} to a_{2i-1}.

Base Case. Since $f \neq id_H$ and H is generated by involutions, we can find $a_0 \neq b_0$ of order 2 in H such that $f(a_0) = b_0$. Let $c_0 = e$ and $K_0 = \{e\}$.
Inductive Case. Let $i > 0$, and suppose that $K_j \leqslant_{fin} H$, $c_j \in H$ and (a_{2j-1}, a_{2j}), $(b_{2j-1}, b_{2j}) \in H^2$ have been defined for every $j < i$. Using Proposition 12, we find commuting $a_{2i-1} \neq b_{2i-1} \in \mathbf{C}_H(K_{i-1}) - K_{i-1}$ of order 2 such that $f(a_{2i-1}) = b_{2i-1}$. Analogously, we find commuting $a_{2i} \neq b_{2i} \in \mathbf{C}_H(\langle K_{i-1}, a_{2i-1}, b_{2i-1}\rangle_H) - \langle K_{i-1}, a_{2i-1}, b_{2i-1}\rangle_H$ of order 2 such that $f(a_{2i}) = b_{2i}$. Then, letting

$$K^* = \langle K_{i-1}, a_{2(i-1)}, a_{2i-1}, a_{2_i}, b_{2(i-1)}, b_{2i-1}, b_{2_i}\rangle_H,$$

$$K^* = K_{i-1} \oplus \langle a_{2(i-1)}, a_{2i-1}, a_{2_i}, b_{2(i-1)}, b_{2i-1}, b_{2_i}\rangle_H \cong K_{i-1} \oplus ((\mathbb{Z}_2)^3 \oplus (\mathbb{Z}_2)^3).$$

Let $\pi \in Aut(K^*)$ be such that π is the identity on K_{i-1} and it maps:

(1) $a_{2(i-1)} \mapsto a_{2(i-1)}$ and $a_{2i-1} \mapsto a_{2_i} \mapsto a_{2i-1}$;
(2) $b_{2(i-1)} \mapsto b_{2i-1} \mapsto b_{2(i-1)}$ and $b_{2_i} \mapsto b_{2_i}$.

Then there exists $c_i \in H$ such that $\Box_{c_i}$ behaves as π on $Aut(K^*)$. Finally, let $K_i = \langle K_{i-1}, a_{2(i-1)}, a_{2i-1}, b_{2(i-1)}, b_{2i-1}, d_{i-1}, c_i\rangle_H$. Then we fulfill the inductive requirements.

Let now, for $i < \omega$, $c_i^* = c_0 \cdots c_i$ and $g = \lim(\Box_{c_i^*} : i < \omega) \in Aut(H)$. Then for every $i < \omega$ we have:

$$a_{2i} \overset{f}{\mapsto} b_{2i} \overset{g}{\mapsto} b_{2i+1} \overset{f^{-1}}{\longmapsto} a_{2i+1} \overset{g^{-1}}{\longmapsto} a_{2i+2},$$

and so $g^{-1}f^{-1}gf$ has infinite order, as wanted. □

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