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Clubs on quasi measurable cardinals

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We construct a model satisfying $\kappa < 2^{\aleph_0} + \clubsuit_\kappa +$ "κ is quasi measurable". Here, we call κ quasi measurable if there is an $\aleph_1$-saturated κ-additive ideal $\mathcal{I}$ on κ. We also show that, in this model, forcing with $\wp(\kappa)\backslash\mathcal{I}$ adds one but not κ Cohen reals. We introduce a weak club principle and use it to show that, consistently, for some $\aleph_1$-saturated κ-additive ideal $\mathcal{I}$ on κ, forcing with $\wp(\kappa)/\mathcal{I}$ adds one but not κ random reals.

1 Introduction

A regular uncountable cardinal κ is called quasi measurable (cf. [1]) if there exists a κ-additive $\aleph_1$-saturated ideal $\mathcal{I}$ on κ. Here, by $\aleph_1$-saturation we mean that there is no uncountable family of pairwise disjoint sets in $\mathcal{I}^+ = \wp(\kappa)\backslash\mathcal{I}$ (equivalently, the quotient algebra $\wp(\kappa)/\mathcal{I}$ is c.c.c.). Examples of such cardinals include real-valued measurable cardinals and Cohen measurable cardinals (cf. [3, 4, 6] for more on such cardinals).

Definition 1.1 For a regular uncountable cardinal κ and a stationary $S \subseteq \kappa$ of limit ordinals, $\clubsuit_S$ is the statement: There exists a sequence $\bar{A} = \langle A_\alpha : \alpha \in S\rangle$ such that each A_α is an unbounded subset of α and for every unbounded $X \subseteq \kappa$, the set $\{\alpha \in S : A_\alpha \subseteq X\}$ is non empty.

Fact 1.2 (Kunen) *Suppose $\kappa \geq 2^{\aleph_0}$ is quasi measurable with a witnessing normal ideal $\mathcal{I}$. Then, for every $\mathcal{I}$-positive $S \subseteq \kappa$, $\clubsuit_S$ holds.*

What happens if $\kappa < 2^{\aleph_0}$? If we start with a measurable cardinal κ and add $> \kappa$ Cohen or random reals, then in the resulting model $\kappa < 2^{\aleph_0}$ and κ is quasi measurable but $\clubsuit_\kappa$ fails. To see this, first note that since the forcing is c.c.c., any potential $\clubsuit_\kappa$-witnessing sequence $\bar{A}$ already appears in an extension W obtained by adding κ Cohen (or random) reals. Hence if $A \in [\kappa]^\kappa$ is coded by a generic sequence of κ Cohen reals (or random reals) over W, then no infinite subset of A can be in W. In particular $\bar{A}$ cannot be a $\clubsuit_\kappa$-sequence. That κ is quasi measurable follows from the fact that quasi measurable cardinals remain quasi measurable in any c.c.c. extension [7].

We show that $\clubsuit_\kappa$ can consistently hold too. Note that, if κ is measurable and $\mathcal{I}$ is a normal prime ideal on κ, then in the canonical inner model $L[\mathcal{I}]$, the hypothesis of Theorem 1.3 holds for every stationary $S \subseteq \kappa$.

On notation: For a regular uncountable cardinal κ and $\vartheta < \kappa$, define $S^\kappa_\vartheta = \{\lambda < \kappa : \mathrm{cf}(\lambda) = \mathrm{cf}(\vartheta)\}$. If $\mathbb{P}, \mathbb{Q}$ are forcing notions, we write $\mathbb{Q} \lessdot \mathbb{P}$ if $\mathbb{Q} \subseteq \mathbb{P}$ and every maximal antichain in $\mathbb{Q}$ is also a maximal antichain in $\mathbb{P}$. In forcing, we use the convention that a larger condition is the stronger one.

Theorem 1.3 *Suppose $V \models$ "κ is measurable, $S \subseteq S^\kappa_\omega = \{\alpha < \kappa : \mathrm{cf}(\alpha) = \omega\}$, S is stationary in κ and $\clubsuit_S$ holds". Then there is a forcing $\mathbb{P}$ such that in $V^{\mathbb{P}}$, $\kappa < 2^{\aleph_0}$, κ is quasi measurable and $\clubsuit_S$ holds.*

Let Coh_λ (or Ran_λ) denote the forcing for adding λ Cohen reals (or random reals). So Coh_λ is the regular open algebra of 2^λ and Ran_λ is the measure algebra of the product measure space 2^λ (with uniform measure on $2 = \{0, 1\}$). In [2], the authors showed the following: Suppose κ is regular uncountable, $\mathcal{I}$ is a κ-additive ideal on κ and $\wp(\kappa)/\mathcal{I}$ is forcing isomorphic to Coh_λ or Ran_λ. Then $\lambda \geq \kappa^+$. In contrast, we show the following.

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Theorem 1.4 *In the model of the previous theorem, there is a κ-additive $\aleph_1$-saturated ideal $\mathcal{I}$ on κ such that* $\mathrm{Coh}_1 \lessdot \wp(\kappa)/\mathcal{I}$ *but not* $\mathrm{Coh}_\kappa \lessdot \wp(\kappa)/\mathcal{I}$.

Finally, we introduce a weak club principle and use it to show the following.

Theorem 1.5 *Suppose κ is measurable, $S = S^\kappa_\omega = \{\delta < \kappa : \mathrm{cf}(\delta) = \omega\}$ and $\clubsuit_S$ holds. Let $\mathbb{P}$ be the finite support iteration of random forcing of length κ. Then in $V^{\mathbb{P}}$, there is a κ-additive $\aleph_1$-saturated ideal $\mathcal{J}$ on κ such that* $\mathrm{Ran}_1 \lessdot \wp(\kappa)/\mathcal{J}$ *but not* $\mathrm{Ran}_\kappa \lessdot \wp(\kappa)/\mathcal{J}$.

2 Getting clubs

Proof of Theorem 1.3. Suppose κ is measurable, $S \subseteq S^\kappa_\omega$ is stationary in κ and $\clubsuit_S$ holds. Put $\lambda = 2^\kappa$. Let $\mathbb{P}$ be a forcing whose conditions are functions p satisfying the following

(a) $\mathrm{dom}(p) \in [\lambda]^{\leq\aleph_0}$.
(b) If $0 \in \mathrm{dom}(p)$, then $p(0) \in {}^{<\omega}\omega_1$.
(c) For each $\alpha \in \mathrm{dom}(p)\setminus\{0\}$, $p(\alpha) \in {}^{<\omega}2$.
(d) For $p, q \in \mathbb{P}$, define $p \geq q$ iff
 (i) $\mathrm{dom}(q) \subseteq \mathrm{dom}(p)$,
 (ii) for every $\alpha \in \mathrm{dom}(q)$, $q(\alpha) \subseteq p(\alpha)$, and
 (iii) $|\{\alpha \in \mathrm{dom}(p) : p(\alpha) \neq q(\alpha)\}| < \aleph_0$.

From now on we assume CH. The next two claims are easily verified.

Claim 2.1 *The partial order $\mathbb{P}$ satisfies the $\aleph_2$-c.c.*

Claim 2.2 *Forcing with $\mathbb{P}$ collapses $\aleph_1$ and preserves all cardinals $\geq \aleph_2$. Also, $V^{\mathbb{P}} \models \kappa < 2^{\aleph_0} = \lambda$.*

Claim 2.3 *Let $\langle A_\alpha : \alpha \in S\rangle$ witness $\clubsuit_S$ in V where each A_α has order type ω. Then $V^{\mathbb{P}} \models \clubsuit_S$ via the same witness.*

Suppose $p \Vdash \mathring{A} \in [\kappa]^\kappa$. Construct $\langle (p_i, \gamma_i) : i < \kappa\rangle$ such that γ_i's are strictly increasing, $p \leq p_i$ and $p_i \Vdash \gamma_i \in \mathring{A}$. Choose $W \in [\kappa]^\kappa$ such that $\langle \mathrm{dom}(p_i) : i \in W\rangle$ forms a delta system with root R and $p_i \restriction R$ does not depend on $i \in W$. Let $X = \{\gamma_i : i \in W\}$. Choose $\alpha \in S$ such that $A_\alpha \subseteq X$. Put $q = \bigcup\{p_i : i \in A_\alpha\}$. Then $q \in \mathbb{P}$, $p \leq q$ and $q \Vdash A_\alpha \subseteq \mathring{A}$.

Claim 2.4 *In $V^{\mathbb{P}}$, κ is quasi measurable as witnessed by the ideal generated by a normal prime ideal on κ.*

This is essentially an argument due to Příkrý [7]. Let $\mathcal{I}$ be a normal prime ideal on κ. Let $\hat{\mathcal{I}} = \{X \subseteq \kappa : (\exists Y \in \mathcal{I})(X \subseteq Y)\}$ be the ideal generated by $\mathcal{I}$ in $V^{\mathbb{P}}$. Suppose $p \Vdash \langle \mathring{X}_i : i < \vartheta\rangle \subseteq \hat{\mathcal{I}}$ where $\vartheta < \kappa$. Since $\mathbb{P}$ satisfies $\aleph_2$-c.c., we can find $\langle Y_i : i < \vartheta\rangle$ such that each $Y_i \in \mathcal{I}$ and $p \Vdash \mathring{X}_i \subseteq Y_i$. Put $Y = \bigcup\{Y_i : i < \vartheta\}$. Then $Y \in \mathcal{I}$ and $p \Vdash \bigcup\{\mathring{X}_i : i < \vartheta\} \subseteq Y$. So $\hat{\mathcal{I}}$ is κ-additive. Next, towards a contradiction, suppose $\hat{\mathcal{I}}$ is not $\aleph_1$-saturated in $V^{\mathbb{P}}$. Since $(\aleph_1)^{V^{\mathbb{P}}} = (\aleph_2)^V$, there exist $p \in \mathbb{P}$ and $\langle \mathring{X}_i : i < \omega_2\rangle \in V^{\mathbb{P}}$ such that $p \Vdash \mathring{X}_i \in \wp(\kappa)\setminus\hat{\mathcal{I}}$ are pairwise disjoint. For each $i < \omega_2$, put $Y_i = \{\alpha < \kappa : (\exists p_{\alpha,i} \geq p)(p_{\alpha,i} \Vdash \alpha \in \mathring{X}_i)\}$. Then $\kappa\setminus Y_i \in \mathcal{I}$ (as $\mathcal{I}$ is prime) hence we can pick some $\alpha \in \bigcap\{Y_i : i < \omega_2\}$. But now the witnesses $\{p_{\alpha,i} : i < \omega_2\}$ are pairwise incompatible: A contradiction.

This concludes the proof of Theorem 1.3. □

Proof of Theorem 1.4. Let $\mathcal{I}$ be a normal prime ideal on κ and $j : V \to M$, the corresponding ultrapower embedding. Let $\mathbb{B}$ be the Boolean completion of $\mathbb{P}$. Let H be $j(\mathbb{B})$-generic over V. Define $G = \{p \in \mathbb{B} : j(p) \in H\}$. Since $\mathbb{B}$ satisfies $\aleph_2$-c.c., G is $\mathbb{B}$-generic over V. So $j[G]$ is $j[\mathbb{B}]$-generic over V. Note also that $j[\mathbb{B}] \lessdot j(\mathbb{B})$. In $V[G]$, let $\hat{\mathcal{I}}$ be the ideal generated by $\mathcal{I}$. Define $\varphi : \wp(\kappa)/\mathcal{I} \to j(\mathbb{B})/j[G]$ by $\varphi(X) = [[\kappa \in j(\mathring{X})]]_{j(\mathbb{B})}/j[G]$.

Claim 2.5 *The function φ is a Boolean isomorphism.*

Suppose $p \Vdash_{\mathbb{B}} \mathring{X} \triangle \mathring{Y} \subseteq A \in \mathcal{I}$. Then $j(p) \Vdash_{j(\mathbb{B})} (\kappa \in j(\mathring{X}) \text{ iff } \kappa \in j(\mathring{Y}))$. Hence $[[\kappa \in j(\mathring{X})]]_{j(\mathbb{B})} \wedge j(p) = [[\kappa \in j(\mathring{Y})]]_{j(\mathbb{B})} \wedge j(p)$ so φ is well defined. It is easily verified that φ preserves Boolean operations. To see that it is injective, note that if $\varphi(X) = 0$, then for some $p \in G$, $[[\kappa \in j(\mathring{X})]]_{j(\mathbb{B})} \cap j(p) = 0$ hence $p \Vdash \mathring{X} \in \hat{\mathcal{I}}$. Finally, if $q \in j(\mathbb{B})/j[G]$, then $q = [\langle p_\alpha : \alpha < \kappa \rangle]$ for some $p_\alpha \in \mathbb{B}$. Let $\mathring{X}$ be such that $[[\alpha \in \mathring{X}]]_{\mathbb{B}} = p_\alpha$. Then $[[\kappa \in j(\mathring{X})]]_{j(\mathbb{B})} = j(\mathring{X})(\kappa) = j(\mathring{X})([\mathrm{id}]) = [\langle p_\alpha : \alpha < \kappa \rangle] = q$. So φ is surjective.

Claim 2.6 *In $V[G]$, forcing with $\wp(\kappa)/\hat{\mathcal{I}}$ adds one but not κ Cohen reals.*

It is clear that any real added by H on a coordinate outside $j[\lambda]$ is Cohen over $V[G]$. Next observe that, arguing as in the proof of Claim 2.3, in $V[H]$, every $\clubsuit_S$-witnessing sequence from V remains so. It follows that $V[H]$ does not contain a Coh_κ-generic over V (since it would code an unbounded $A \subseteq \kappa$ which would be counterexample to the old $\clubsuit_S$-witnessing sequence) and so $\wp(\kappa)\backslash\hat{\mathcal{I}}$ cannot add one.

This concludes the proof of Theorem 1.4. □

3 A club principle

Definition 3.1 Let κ be a regular uncountable cardinal, $S \subseteq S^\kappa_\omega$ stationary in κ, and $\bar{A} = \langle A_\delta : \delta \in S \rangle$. We say that $\clubsuit^{\sup}_S(\bar{A})$ holds if each $A_\delta = \{\alpha_{\delta,j} : j < \omega\}$ for some sequences $\alpha_{\delta,j}$ strictly increasing with j and cofinal in δ and for every $A \in [\kappa]^\kappa$ and $\varepsilon > 0$, there exists some (equivalently, stationary many) $\delta \in S$ such that

$$\limsup_n \frac{|A \cap \{\alpha_{\delta,j} : j < n\}|}{n} \geq 1 - \varepsilon.$$

Then we say that $\clubsuit^{\sup}_S$ holds if there is some $\bar{A} = \langle A_\delta : \delta \in S \rangle$ such that $\clubsuit^{\sup}_S(\bar{A})$ holds.

Does $\clubsuit^{\sup}_S$ imply $\clubsuit_S$? A negative answer follows from Lemma 3.3 below or just add κ^+ Cohen reals.

Claim 3.2 *Suppose $\kappa = \mathrm{cf}(\kappa) \geq \aleph_1$, $S \subseteq S^\kappa_\omega$ is stationary and $\bar{A} = \langle A_\delta : \delta \in S \rangle$ where each A_δ is an unbounded subset of δ of order type ω. Then $V^{\mathrm{Ran}_\kappa} \models \neg\clubsuit^{\sup}_S(\bar{A})$.*

P r o o f. Let $r \in 2^\kappa$ be Ran_κ-generic over V. Take $A = \{\alpha < \kappa : r(\alpha) = 1\}$, $\varepsilon < 0.5$ and use the fact that whenever $\{\alpha_j : j < \omega\} \in [\kappa]^{\aleph_0} \cap V$, we have

$$\lim_n \frac{|\{j < n : r(\alpha_j) = 1\}|}{n} = 0.5. \qquad \square$$

Lemma 3.3 *Suppose κ is regular uncountable, $S \subseteq S^\kappa_\omega$ is stationary in κ, $\bar{A} = \langle A_\delta : \delta \in S \rangle$ and $\clubsuit^{\sup}_S(\bar{A})$ holds. Let $\mathbb{P}$ be the finite support iteration of random forcing of length ϑ where ϑ is any ordinal. Then $V^{\mathbb{P}} \models \clubsuit^{\sup}_S(\bar{A})$.*

P r o o f. Suppose $p_\star \Vdash \mathring{A} \in [\kappa]^\kappa$ and $\varepsilon_\star > 0$. We'll find an $r \geq p_\star$, $\delta \in S$ witnessing $\clubsuit^{\sup}_S(\bar{A})$ for $\mathring{A}$. Let $A' = \{\alpha < \kappa : (\exists p_\alpha \geq p_\star)(p_\alpha \Vdash \alpha \in \mathring{A})\}$. For each $\alpha \in A'$, choose $p_\alpha \geq p_\star$ such that $p_\alpha \Vdash \alpha \in \mathring{A}$. The following claim is easy to verify.

Claim 3.4 *Given $\varepsilon > 0$, $q \in \mathbb{P}$, there exist $r \geq q$, $\bar{\sigma} = \langle \sigma_\xi : \xi \in dom(r) \rangle$, $\bar{\varepsilon} = \langle \varepsilon_\xi : \xi \in dom(r) \rangle$ such that for every $\xi \in dom(r)$, $\sigma_\xi \in {}^{<\omega}2$, $0 < \varepsilon_\xi < \varepsilon/2^{|dom(r)\backslash\xi|+2}$, ε_ξ is rational and $r{\restriction}\xi \Vdash_{\mathbb{P}_\xi} \mu([\sigma_\xi] \cap r(\xi)) \geq (1 - \varepsilon_\xi)\mu([\sigma_\xi])$.*

For each $\alpha \in A'$, let $r_\alpha, \bar{\sigma}_\alpha, \bar{\varepsilon}_\alpha$ be as in previous claim w.r.t. $q = p_\alpha$, $\varepsilon = \varepsilon_\star$. Choose $A'' \in [A']^\kappa$ such that the following hold.

(a) For each $\alpha \in A''$, $|\mathrm{dom}(r_\alpha)| = n_\star$ is constant and $\langle \mathrm{dom}(r_\alpha) : \alpha \in A'' \rangle$ forms a delta system, $\mathrm{dom}(r_\alpha) = \{\xi_{\alpha,0} < \xi_{\alpha,1} < \cdots < \xi_{\alpha,n_\star-1}\}$, $v \subseteq n_\star$ and $\{\xi_{\alpha,i} : i \in v\} = \{\xi_k : k < |v|\}$ is the root,

(b) $\sigma_{\alpha,\xi_k} = \sigma_k$ and $\varepsilon_{\alpha,\xi_k} = \varepsilon_k$, for $k < |v|$, do not depend on $\alpha \in A''$.

Choose $\delta \in S$ such that

$$\limsup_n \left\{ \frac{|A_\delta \cap A'' \cap \alpha|}{n} : \alpha < \delta \wedge |A_\delta \cap \alpha| = n \right\} > 1 - \frac{\varepsilon_\star}{2}.$$

Claim 3.5 *There exists* $r \geq p_\star$ *such that*

$$r \Vdash \limsup_n \left\{ \frac{|A_\delta \cap \mathring{A} \cap \alpha|}{n} : \alpha < \delta \wedge |A_\delta \cap \alpha| = n \right\} \geq 1 - \varepsilon_\star.$$

Let $\{\alpha_n : n < \omega\}$ be an increasing enumeration of A_δ. Put

$$W = \left\{ n : \frac{|A'' \cap \{\alpha_i : i < n\}|}{n} \geq 1 - \frac{\varepsilon_\star}{2} \right\}.$$

For each $\xi \leq \vartheta$, let

$$\mathring{W}_\xi = \left\{ n \in W : \frac{|\{i < n : \alpha_i \in A'' \wedge r_{\alpha_i} \restriction \xi \in G_{\mathbb{P}_\xi}\}|}{n} \geq 1 - (\varepsilon_\star/2 + \sum_{j \in v, \xi_j < \xi} 2\varepsilon_j) \right\}.$$

It is clearly enough to find $r \geq p_\star$ such that the following hold.

(i) $\mathrm{dom}(r) = \{\xi_k : k < |v|\}$,
(ii) For every $\xi \in \mathrm{dom}(r) \cup \{\vartheta\}$, $r \restriction \xi \Vdash_{\mathbb{P}_\xi} \mathring{W}_\xi$ is infinite.

Recursively, construct $r \restriction \xi$ as follows. For $\xi \leq \xi_0$, $r \restriction \xi$ is the empty condition. Given $r \restriction \xi_k$, define $r \restriction \xi_{k+1}$ using the following.

Claim 3.6 *Suppose* $0 < \varepsilon < 1/2$, $W' \in [\omega]^\omega$, $\sigma \in {}^{<\omega}2$, $\langle A_n : n < \omega \rangle$ *are Borel satisfying* $\mu(A_n \cap [\sigma]) \geq (1-\varepsilon)\mu([\sigma])$. *Let* $\mathbb{Q} = \mathrm{Ran}$. *Then there exists some* $A \in \mathbb{Q}$ *such that* $\mu(A \cap [\sigma]) \geq \mu([\sigma])/2$ *and*

$$A \Vdash_{\mathbb{Q}} \limsup_{n \in W'} |\{j < n : A_j \in G_{\mathbb{Q}}\}|/n \geq 1 - 2\varepsilon.$$

Without loss of generality, $\sigma = \varphi$. Let $f_n = (1/n)\sum_{j<n} 1_{A_j}$. Then $\int \limsup_{n \in W'} f_n \geq \limsup_{n \in W'} \int f_n \geq 1 - \varepsilon$. Let $A = \{x : \limsup_{n \in W'} f_n(x) \geq 1 - 2\varepsilon\}$.

This concludes the proof of Lemma 3.3. □

Proof of Theorem 1.5. Choose $\bar{A}$ such that $\clubsuit_S^{\sup}(\bar{A})$ holds. Let $\mathcal{I}$ be a normal prime ideal on κ. Let $j : V \to M$ be the ultrapower embedding (using $\mathcal{I}$) with critical point κ. $j(\mathbb{P})$ is a finite support iteration of random forcing of length $j(\kappa)$. Let G be $\mathbb{P}$-generic over V. In $V[G]$, let $\mathcal{J}$ be the ideal generated by $\mathcal{I}$. Then $\wp(\kappa)/\mathcal{J}$ is forcing equivalent to $j(\mathbb{P})/G$ which, in $M[G]$, is the finite support iteration of random forcing indexed by $j(\kappa)\backslash\kappa$—cf., e.g., [5] for such arguments. Since $M[G]$ and $V[G]$ have same reals, $\mathrm{Ran}_1 \lessdot \wp(\kappa)/\mathcal{J}$. This forcing doesn't add κ random reals since it completely embeds into $j(\mathbb{P})$ which, by Lemma 3.3, preserves $\clubsuit_S^{\sup}(\bar{A})$ whereas Ran_κ destroys it (Claim 3.2). □

4 Final remarks

For any $\vartheta < \kappa$, we can improve Theorem 1.4 to get $\mathrm{Coh}_\vartheta \lessdot \wp(\kappa)/\mathcal{I}$ by modifying our forcing as follows. Let $\mathbb{P}$ be as in the proof of Theorem 1.3. For $\vartheta < \kappa$, define $\mathbb{P}_\vartheta = \{p \in \mathbb{P} : (\forall \alpha < \kappa)(|\mathrm{dom}(p) \cap [\vartheta\alpha, \vartheta(\alpha+1))| < \aleph_0)\}$ ordered as before. Then in $V^{\mathbb{P}_\vartheta}$, letting $\mathcal{I}$ to be the ideal generated by a normal prime ideal on κ, we have that $\mathrm{Coh}_\vartheta \lessdot \wp(\kappa)/\mathcal{I}$ but not $\mathrm{Coh}_\kappa \lessdot \wp(\kappa)/\mathcal{I}$. Similarly for Theorem 1.5 using a finite support iteration of Ran_ϑ. But what about the following?

Question 4.1 Is it consistent to have a κ-additive $\aleph_1$-saturated ideal $\mathcal{I}$ on κ such that for every $\vartheta < \kappa$, $\mathrm{Coh}_\vartheta \lessdot \wp(\kappa)/\mathcal{I}$ but not $\mathrm{Coh}_\kappa \lessdot \wp(\kappa)/\mathcal{I}$? Similarly for random forcing.

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