PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 119, Number 1, September 1993

ADDING DOMINATING REALS WITH THE RANDOM ALGEBRA

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(Communicated by Andreas R. Blass)

ABSTRACT. We show that there are two models $M \subseteq N$ such that by forcing with $(\text{Random})^M$ over N we add dominating reals. This answers a question of A. Miller.

Let R be the random real forcing. It is well known that R is an ω^ω-bounding forcing notion, that is

$$(\forall f \in \omega^\omega \cap V^R \exists g \in \omega^\omega \cap V)(\forall n \in \omega)(f(n) < g(n)).$$

For more detail and notation the reader should see [Ku]. The following is also known [BJ2]:

In $V^{R\times R}$ there are Cohen reals over V.

From this we can conclude the following.

There are models $M \subset N$ such that in $N^{R\cap M}$ there are unbounded reals over N. (Take $N = M^R$ and use the previous result.)

After this it was natural to ask:

(1) Are there $M \subseteq N$ such that in $N^{R\cap M}$ there are dominating reals over N?

Let us introduce more notation. Let I be an ideal of subsets of $\mathbf{R}$. Then we define $K_A(I)$ as the cardinality of the smallest family of elements of I whose union is not in I. $K_B(I)$ is the cardinality of the smallest covering of the reals by elements of I. b is the cardinality of the smallest family of functions from ω to ω, which is unbounded. Miller [Mi] proved that

$$\text{cof}(K_B(\text{Meager})) > \omega.$$

More generally, Bartoszynski and Judah [BJ1] proved that

$$\text{cof}(K_B(\text{Meager})) \geq K_A(\text{Measure zero}).$$

It is an open problem if

$$\text{cof}(K_B(\text{Meager})) \geq K_A(\text{Meager}).$$

Received by the editors November 20, 1989 and, in revised form, January 14, 1992.
1991 *Mathematics Subject Classification*. Primary 03E05; Secondary 04A20.
The first author (J. Ihoda) would like to thank NSF under Grants DMS-8505550 and MSRI for partial support.
The second author would like to thank BSF, NSF, and MSRI for partial support.

After the Miller result it was natural to ask

$$\text{Is } \operatorname{cof}(K_B(\text{Measure zero})) > \omega?$$

This question remains open and has produced a lot of development. The only positive result in this direction is the following theorem of Bartoszynski [Ba]:

$$\operatorname{cof}(K_B(\text{measure zero})) > \omega \quad \text{if } b \geq K_B \text{ (measure zero)}.$$

When we started working on this problem we proposed the iteration

$$\overline{Q} = \langle P_\alpha, \mathbf{Q}_\beta : \alpha \geq \aleph_{\omega+1}, \beta < \aleph_{\omega+1} \rangle$$

satisfying

(i) $\vDash$ "$\mathbf{Q}_0$ adds $\aleph_\omega$-many Cohen reals",
(ii) $\vDash_{P_\beta}$ "$\mathbf{Q}_\beta$ is a subalgebra of Random reals of cardinality less than $\aleph_\omega$";
(iii) The sequence is generic enough in order to force with every possible subalgebra of Random reals.

Our conjecture was

$$V^{P_{\aleph_{\omega+1}}} \Vdash \text{“}K_B \text{ (measure zero)} = \aleph_\omega\text{”}.$$

After hard work we started thinking that maybe we were missing something and we asked:

$$\text{Maybe } V^{P_{\aleph_{\omega+1}}} \Vdash b \geq \aleph_\omega?$$

It is easy to see (by [Ba]) that this is true if for some $\beta < \aleph_{\omega+1}$

$$\Vdash_{P_\beta} \text{“}\mathbf{Q}_\beta \text{ adds a dominating real”}.$$

Therefore the question was: Does there exist $R' \subseteq R$ such that

$$\Vdash_{R'} \text{“add a dominating real”?}$$

We show in [JS2] that under CH, or under $K_B(\text{Meager}) = 2^{\aleph_0}$, there exists such subalgebras of R. And using this and Bartoszynski's result, it is not hard to see that

$$V^{P_{\aleph_{\omega+1}}} \vDash K_B(\text{measure zero}) = \aleph_{\omega+1}.$$

The construction in [JS2] was not strong enough to solve Miller's question (1). That is, our example was not the random algebra restricted to some inner model; therefore, we thought that we could change condition (ii) of the iteration to

(ii)* $\Vdash_{P_\beta}$ "$\mathbf{Q}_\beta = M \cap R$ for some inner model $M \vDash 2^{\aleph_0} = \aleph_n$, for $n < \omega$".

Again we were unable to show that this new iteration gives the desired model, and we recalled Miller's question (1).

In this work we will answer Miller's question (1) positively by showing:

There are two models $M \subseteq N$ such that forcing over N with $R \cap M$ we add dominating reals. We sketch the construction as follows: We start for simplicity with $V = L$. Then we add $\aleph_2$ Cohen reals. After this we add, with finite support iteration, a sequence $\langle r_i : i < \omega_1 \rangle$ of positive sets by a forcing notion, which is Souslin (see [JS1]) and has the appearance of Amoeba forcing. Then we let $M = V[\langle r_i : i < \omega_1 \rangle]$ and $N = V[\aleph_2 - \text{Cohen}][\langle r_i : i < \omega \rangle]$.

The notation is standard and the rest of the paper is devoted to building the models.

1. **Assumption.** Let $\overline{W} = \langle W_n : n < \omega \rangle$ be a sequence of pairwise disjoint subsets of ω. We also assume

(0) W_n is infinite.

(1) For every $n < \omega$, $m < \omega$, and $k < \omega$ there are $i < j < \omega$ such that

(a) $\bigcup_{l \neq n} W_l \cap [i, j] = \varnothing$;

(b) $2^{2^{2^{2^j}}} < \min[\bigcup_{l \neq n} W_l \setminus [0, i)]$;

(c) $2^{-2^i} \cdot \frac{1}{k} > \sum\{2^{-l} : l \in W_n \cap [i, j]\}$;

(d) There is $u \subseteq W_n \cap [i, j]$ satisfying $|u| > [2^{2^{2^{2^i}}}]^{m+1}$;

(e) $i > k$.

2. **Definition.** (i) $X \subseteq {}^{\omega >}2$ *obeys* $u \subseteq \omega$ if there exist $u_X \subseteq u$, $\rho_n \in {}^n 2$ such that $X = \{\rho_n : n \in u_X\}$.

(ii) X *almost obeys* u if it obeys $u \cup \{0, 1, \dots, n\}$ for some n.

(iii) $({}^\omega 2)^{[X]} = \bigcup_{\rho \in X} ({}^\omega 2)^{[\rho]} = \{\nu : \nu \in {}^\omega 2 \wedge (\exists \rho)(\rho \in X \wedge \rho \subset \nu\}$.

(iv) X obeys $\overline{W}$ if it almost obeys each $\bigcup_{l > n} W_l$.

Let $L_b M_s$ denote Lebesgue measure on 2^ω.

3. **Definition.** $Q = Q(\overline{W})$ is defined as follows:

(I) A condition has the form (t, X) where

(i) t is a function from ${}^{n \geq}2$ to $\mathbb{Q} \cap [0, 1)$ and $2^{4|\rho|} \cdot t(\rho)$ is an integer for all $\rho \in {}^{n \geq}2$;

(ii) $t(\eta) = t(\eta \wedge \langle 0 \rangle) + t(\eta \wedge \langle 1 \rangle)$ for $\eta \in {}^{n >}2$;

(iii) for $\rho \in {}^n 2$, $t(\rho) < L_b M_s(({}^\omega 2)^{[\rho]} \setminus ({}^\omega 2)^{[X]})$;

(iv) X is a finite union of sets obeying $\overline{W}$.

(II) $(t_1, X_1) \leq (t_2, X_2)$ iff $t_1 \subseteq t_2$, $X_1 \subset X_2$, and if $\eta \in \mathrm{Dom}(t_2) \setminus \mathrm{Dom}(t_1)$ and $\eta \in X_1$, then $t_2(\eta) = 0$.

4. *Claim.* $Q \models$ Souslin.

5. *Claim.* $Q \models$ ccc.

Proof. Let $\langle \langle t_\alpha, X_\alpha \rangle : \alpha < \omega_1 \rangle$ be an ω_1-sequence of members of Q. W.l.o.g. $t_\alpha = t_\beta$ for $\alpha \neq \beta < \omega_1$. Now let r_α^ρ be a positive rational satisfying

$$r_\alpha^\rho < t_\alpha(\rho) - L_b M_s(({}^\omega 2)^{[\rho]} \setminus ({}^\omega 2)^{[X]}).$$

Therefore w.l.o.g. $r_\alpha^\rho = r_\beta^\rho$ for $\alpha \neq \beta < \omega_1$. Let m_ρ be such that $2^{-m_\rho} < r_\alpha^\rho / 4^{|t|}$. Let $m = \max\{m_\rho : \rho \in \mathrm{Dom}(t_\alpha)\}$. Also w.l.o.g. we may assume that $X_\alpha \restriction m = X_\beta \restriction m$. Then it is not hard to see that $\langle \langle t_\alpha, X_\alpha \rangle : \alpha < \omega_1 \rangle$ is a set of pairwise compatible members of Q. □

6. *Notation.* Let $\mathrm{Dom}^+(t) = \{\eta \in \mathrm{Dom}(t) : t(\eta) > 0\}$. $\mathbf{T} = \bigcup\{\mathrm{Dom}^+(t) : \langle t, X \rangle \in \mathbf{G}_Q\}$ is a Q-name.

7. *Claim.* If X_n obeys W_n for $n < \omega$, each X_n finite $\langle X_n : n < \omega \rangle \in V$, then in V^Q the following hold:

(i) $\mathbf{T}$ is a perfect subset of ${}^{\omega >}2$.

(ii) $L_b M_s(\lim \mathbf{T}) = t\langle 0 \rangle$ for some (any) $\langle t, X \rangle \in G_Q$.

(iii) For some k, $\mathbf{T}$ is disjoint from $\bigcup_{n \geq k} X_n$.

(iv) Therefore, $\lim(\mathbf{T}) \cap ({}^\omega 2)^{[\bigcup_{n > k} X_n]} = \varnothing$.

Proof. Clear. □

Now we will introduce a technical device that we will use in order to build our models. After this we will show a theorem about finite support iteration forcing.

8. **Definition.** Let λ be a cardinal, and n an integer. We call $\overline{X}$ (λ, n)-*big* if
(i) $\overline{X}$ is a family of subsets of ${}^{\omega>}2$ each one obeying $\overline{W}$;
(ii) If $X_\zeta \in \overline{X}$ for $\zeta < \lambda$ are pairwise distinct then for every m, k there are $[i, j]$ and u such that
(a) $\bigcup_{l\neq n} W_l \cap [i, j] = \varnothing$;
(b) $u \subseteq W_n \cap [i, j]$;
(c) $|u| > [2^{2^{2^{i}}}]^{m+1}$;
(d) $\min(\bigcup_{l\neq n} W_l \backslash [0, i]) > 2^{2^{2^{j}}}$;
(e) $2^{-2^i} \cdot \frac{1}{k} > \sum_{l\in W_n\cap[i,j]} 2^{-l}$;
(f) for every $\langle \rho_l : l \in u\rangle \in \prod_{l\in u} {}^l2$ there is ζ such that for each $l \in u$, $X_\zeta \cap {}^l2 = \{\rho_l\}$;
(g) $i > k$.

9. **Lemma.** *Assume that*
(i) $\overline{Q} = \langle P_i, Q_j : i \leq \alpha,\ j < \alpha\rangle$ *is a finite support iteration and* $\mathrm{cof}(\lambda) = \lambda > \aleph_0$, $\lambda > \alpha = \bigcup \alpha \neq 0$.
(ii) $\overline{X}$ *is* (λ, n)-*big in* V.
(iii) *For each* $i < \alpha$, $\overline{X}$ *is* (λ, n)-*big in* V^{P_i}.
Then in V^{P_α} *we also have that* $\overline{X}$ *is* (λ, n)-*big*.

Proof. Trivial $(\lambda > \alpha!)$. □

10. **Lemma.** Q *preserves* "$\overline{X}$ *is* (λ, n)-*big*" *when* $\mathrm{cof}(\lambda) = \lambda > \aleph_0$.

Proof. Suppose that $p \Vdash_Q$ "$\langle \mathbf{X}_\zeta : \zeta < \lambda\rangle$ is a counterexample". Choose by induction on $\beta < \lambda$, $\zeta_\beta < \lambda$, $\mathbf{X}_{\zeta_\beta}$, X_β, p_β such that $p \leq p_\beta \in Q$

$$p_\beta \Vdash \text{“}\mathbf{X}_{\zeta_\beta} = X_\beta \wedge X_\beta \notin \{X_\gamma : \gamma < \beta\}\text{”}.$$

Let $p_\beta = \langle t_\beta, Y_\beta\rangle$, w.l.o.g. $t_\beta = t$, $\mathrm{Dom}(t_\beta) = {}^{n^*\geq}2 Y_\beta = \bigcup_{l<k^*} Y_{\beta,l}$, and each $Y_{\beta,l}\backslash {}^{e\geq}2$ obey $\bigcup_{l>n} W_l$. Let $k^1 > k^*$, k such that $4n^* < k^1$, and let $m^1 = m+1$. We know that $\overline{X}$ is (λ, n)-big in V; therefore, we can find $[i, j]$, n, and $\langle \zeta(\overline{\rho}) : \overline{\rho} \in \prod_{l\in n} {}^l2\rangle$ satisfying conditions 8(i), (ii) for k^1, m^1.

11. *Claim.* There is $u_1 \subseteq u$ such that $|u_1| \geq (2^{2^{2^{l}}})^{m+1}$ and there is a function $H : \prod_{l\in u_1} {}^l2 \to \prod_{l\in u} {}^l2$ satisfying
(a) $(H(\overline{\rho}))|u_1 = \overline{\rho}$;
(b) for every $\overline{\rho}_1 \overline{\rho}_2 \in \prod_{l\in u_1} {}^l2$ we have

$$\langle Y_{\zeta(H(\overline{\rho}_1)),l} \cap {}^{i>}2 : l < k^*\rangle = \langle Y_{\zeta(H(\overline{\rho}_2)),l} \cap {}^{i>}2 : l < k^*\rangle.$$

Proof. We know that $i > k^1 > k^*$; therefore, $2^{2^{2^{l}}}$ is bigger than the number of possibles $\langle Y_{\zeta,l} \cap {}^{i>}2 : l < k^*\rangle$. This means that the function $G : \prod_{l\in u} {}^l2 \to \mathrm{Range}(G)$, given by

$$G(\overline{\rho}) = \langle Y_{\zeta(\overline{\rho}),l} \cap {}^{i>}2 : i < k^*\rangle,$$

satisfies $|\mathrm{Range}(\overline{\rho})| \le 2^{2^{2^{2^i}}}$. On the other hand we have $|u| > (2^{2^{2^{2^i}}})^{m^1+1}$ and $m^1 = m+1$. From this we have that there are a sequence of disjoint sets $\langle u_e : e \in \mathrm{Range}(G)\rangle$, each $u_e \subseteq u$ satisfying

$$|u_e| \ge (2^{2^{2^{2^i}}})^{m+1}.$$

Fix an ordering of $\mathrm{Range}(G)$. By induction on $e \in \mathrm{Range}(G)$ we will try to pick $\overline{\rho}^e$ satisfying

(i) $\overline{\rho}^e \in \prod_{l \in u_e} {}^l 2$;

(ii) If $\overline{\rho} \in \prod_{l \in u} {}^l 2$ and $\overline{\rho}^e = \overline{\rho} \restriction u_e$ and for each $e^1 < e$ $\overline{\rho}^{e^1} = \overline{\rho} \restriction u_{e^1}$, then $G(\overline{\rho}) \ne e$. If we can do this induction then let $\rho^* = \bigcup_{e \in \mathrm{Range}(G)} \overline{\rho}^e$, and clearly $G(\rho^*) \ne e$ for every $e \in \mathrm{Range}(G)$—a contradiction.

Therefore there exists the first $e \in \mathrm{Range}(G)$ such that we cannot pick $\overline{\rho}^e$ satisfying (i) and (ii). This means that for each $\overline{\rho} \in \prod_{l \in u_e} {}^l 2$ there is $v_\rho \in \prod_{l \in u} {}^l 2$, $\rho = v_\rho \restriction u_l$, and $G(v_\rho) = e$. This clearly defines H. □

Now we will finish with the proof of Lemma 10.

Let u_1, H be given by the claim. It will be enough to show that

$$\langle t, X\rangle = \left\langle t, \bigcup_{\substack{l<k^* \\ \overline{\rho} \in u_1}} Y_{\zeta(H(\overline{\rho})), l} \right\rangle$$

is a condition.

By assumption on u_1 and H

$$X \cap {}^{i>}2 = \bigcup_{l<k^*} Y_{\zeta(H(\overline{\rho})), l}$$

(for some (any) $\rho \in \prod_{l \in u_1} {}^l 2$).

We should check only 3(I)(iii). We know that for each $\eta \in t$

$$(\alpha) \qquad t(\eta) > 2^{-4n^*}.$$

Also we know that

$$(\beta) \qquad t(\eta) - L_b M_s(({}^\omega 2)^{[\eta]} \setminus ({}^\omega 2)^{[X]}) > 0;$$

therefore, for each $\rho \in X \cap {}^{i>}2$ and ρ compatible with η

$$(\gamma) \qquad t(\eta) - 2^{-|\rho|} > 0,$$

$$(\delta) \qquad 2^{i-1}(t(\eta) - 2^{-|\rho|}) \text{ is an integer.}$$

Therefore

$$t(\eta) - L_b M_s(({}^\omega 2)^{[\eta]} \setminus ({}^\omega 2)^{[X \cap {}^{i>}2]}) > 2^{-(i-1)}.$$

Now by assumption 8(e)

$$L_b M_s(({}^\omega 2)^{[\eta]} \cap ({}^\omega 2)^{[X \cap {}^{[i,j)}2]}) < 2^{-2^i}$$

and also

$$L_b M_l(({}^\omega 2)^{[\eta]} \cap ({}^\omega 2)^{[X \cap {}^{\omega - j}2]}) < 2^{-j}.$$

Thus

$$t(\eta) - L_b M_s(({}^{\omega}2)^{[X]} \cap ({}^{\omega}2)^{[\eta]}) > 0. \quad \square$$

Let $V \vDash CH$, and let $\overline{W} = \langle W_n : n < \omega \rangle$ satisfy the condition from 2, each W is infinite. Let $\overline{Q} = \langle P_i, Q_j : i \leq \omega_1, \ j < \omega_1 \rangle$ be a finite support iteration. Q_0 adds $\aleph_2$ Cohen reals. We interpret it as adding $\langle X_{\zeta,n} : \zeta < \omega_2, \ n < \omega \rangle$, where $X_{\zeta,n} \subseteq {}^{\omega>}2$ obeys W_n. Each Q_{i+1} is Q from Definition 3.

12. *Claim.* For each $n < \omega$

$$V^{Q_0} \Vdash \text{“}\langle X_{\zeta,n} : \zeta < \omega_2 \rangle \text{ is } \langle \aleph_2, n \rangle\text{-big”}.$$

Proof. Clear using the properties of $\overline{W}$. $\square$

Therefore $V^{P_{\omega_2}} \vDash$ “$\langle X_{\zeta,n} : \zeta < \omega_1 \rangle$ is $(\aleph_2, n)$-big for each n”. Let $\langle T_{i+1} : i < \omega_1 \rangle$ be the reals (perfect trees) given by the Q_i's. Clearly $\langle T_{i+1} : i < \omega_1 \rangle$ is generic for an ω_1-iteration of Q. This is an ω_1-iteration of Souslin forcing satisfying ccc; therefore, $\langle T_{i+1} : i < \omega_1 \rangle$ is also generic over V. Let

$$V^a = V[\langle T_{i+1} : i < \omega_1 \rangle],$$
$$V^b = V[\langle X_{\zeta,n} : \zeta < \omega_2, \ n < \omega \rangle][\langle T_{i+1} : i < \omega_1 \rangle].$$

Then

$$V^a \vDash CH, \qquad V^a \text{ is a class of } V^b.$$

For each $\zeta < \aleph_2$, $X_{\zeta,n} \subseteq {}^{\omega>}2$, so $I_{\zeta,n} = \{({}^{\omega}2)^{[\rho]} : \rho \in X_{\zeta,n}\}$ is a subset of $(\text{Random})^{V^a}$ (but not in V^a!!).

We want to show that “for every large enough ζ, $I_{\zeta,n}$ is a predense subset of $(\text{Random})^{V^a}$”.

If $I_{\zeta,n}$ is a counterexample, then there are $\varepsilon > 0$ and perfect $T_{\zeta,n} \subseteq {}^{\omega}2$ in V^a, such that $(\lim T_{\zeta,n}) \cap \bigcup I_{\zeta,n} = 0$ and $L_b M_s(\lim T_{\zeta,n}) > \varepsilon$. But $V^a \vDash CH$, so for some T, $u = \{\zeta < \aleph_2 : T_{\zeta,n}$ is well defined and $= T\}$ has cardinality $\aleph_2$.

13. *Claim.* $\{X_{\zeta,n} : \zeta \in u\}$ contradicts $\{X_{\zeta,\eta} : \zeta < \omega_2\}$ is $(\aleph_2, n)$-big.

Proof. Clearly $\bigcup_{\zeta \in \eta} X_{\zeta,n}$ contains ${}^{l}2$ for arbitrarily large l (by using 8(e)). Therefore $T \cap {}^{l}2 = \varnothing$. But this contradicts $L_b M_s(\lim T) > 0$. $\square$

So for each $n < \omega$ for every large enough ζ, $I_{\zeta,n}$ is predense in $(\text{Random})^{V^a}$. So for some ζ this holds for every n (really by homogeneity of forcing this holds for ever ζ and we can use Q_0 being one Cohen real).

Let $h_n : I_{\zeta,n} \to \omega$ be such that $h_n(({}^{\omega}2)^{[\rho_1]}) = h_n(({}^{\omega}2)^{[\rho_2]})$ iff ρ_1 and ρ_2 are comparable. So $\langle h_n : n < \omega \rangle$ describes a $(\text{Random})^{V_a}$-name $\mathbf{h}$, namely, $\mathbf{h}(n) = h_n(({}^{\omega}2)^{[\rho]})$ if $({}^{\omega}2)^{[\rho]} \in I_{\zeta,n}$ and $({}^{\omega}2)^{[\rho]} \in$ Generic set. This will be the name of generic real.

Let $B_0 \in (\text{Random})^{V^a}$, $f : \omega \to \omega$ in V^b. We want to prove that for some B_1, $B_0 \subset B_1 \in (\text{Random})^{V^a}$ and $B_1 \Vdash f <^* \mathbf{h}$. Let $Y_n = \{\rho \in X_{\zeta,n} : \rho$ minimal in $X_{\zeta,n}$ (i.e., $\rho \restriction l \notin X_{\zeta,n}$ for $l < lg(l)$) and $h_n(({}^{\omega}2)^{[\rho]}) \leq f(n)\}$. Clearly Y_n is finite and obeys W_n $(Y_n \subseteq X_{\zeta,n})$. Therefore $\bigcup_{n<\omega} Y_n$ almost obeys each $\bigcup_{n>k} W_n$ for each k. Hence it obeys $\overline{W}$.

Also $\bigcup_{n<\omega} Y_n \in V^{P_{\omega_1}}$, so for some $i < \omega_1$, it belongs to V^{P_i}, w.l.o.g. $i > 0$. Now for some $\rho \in {}^{\omega>}2$, $L_b M_s(({}^{\omega}2)^{[\rho]} \cap B_0) \geq 2^{-|\rho|}(1 - 1/100)$.

On the other hand, by genericity we can find $j \in [i, \omega_1)$ such that for some k, $t(\rho) = (1 - 1/100)2^{-|\rho|}$ and (t, ϕ) is in the generic subset for Q_j. Therefore T_{j+1} satisfies $L_b M_s(\lim T_{j+1} \cap ({}^{\omega}2)^{[\rho]}) = (1 - 1/100)2^{-|\rho|}$. So $B_1 = B_0 \cap \lim T_{j+1} \geq B_0$ is a condition in Random V^a (because $T_{j+1} \in V^a$). But it is also forced that for some n, $T_i \cap (\bigcup_{k>n} Y_h) = \varnothing$ by 7(iii). And this implies $B_1 \Vdash f <^* \mathbf{h}$. This finishes the proof of the theorem.

Remark. Recently, Janusz Pawlikowski, motivated by this present work, showed that if you adjoin an "infinitely often equal" real and then force with the random algebra of the ground model, you get dominating reals (in fact Hechler-generic).

REFERENCES

[Ba] T. Bartoszynski, *Covering the real line by measure zero sets*, Pacific J. Math. **1** (1988), 1–12.

[BJ1] T. Bartoszynski and H. Judah, *On cofinality of the smallest covering of the real line by meager sets*, J. Symbolic Logic **54** (1989), 719–726.

[BJ2] ——, *Measure and category in set theory: The assymetry* (in preparation).

[JS1] H. Judah and S. Shelah, *Souslin forcing*, J. Symbolic Logic **53** (1988), 1188–1207.

[JS2] ——, *Around random algebra*, Arch. Math. Logic **30** (1990), 128–138.

[Ku] K. Kunen, *Set theory*, North-Holland, Amsterdam.

[Mi] A. Miller, *Baire theorem and cardinals of countable cofinality*, J. Symbolic Logic **47** (1982).

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