Foreword

You may have wondered: does not the shoemaker go barefoot? Mathematics boasts of being the epitome of exactness, but what is the exact meaning of proof? Construction? Computation?
Or you may be very ambitious and wonder whether we can prove theorems concerning the collection of all possible mathematical theories.
Or you may have resigned yourself to having no exact answer, as you cannot "pull yourself out of the mud," at most you can philosophize about it.
Or you may wonder: is mathematics one body or is it fragmented into many branches; i.e. can we put it all in one framework?
Or you may be philosophically inclined and wonder whether having a proof and being true are the same.

However there is a branch of mathematics dealing exactly with those problems: LOGIC. It is one of the oldest intellectual disciplines (see Aristotle) yet also one which has developed enormously in this century.

Yes! Mathematics can deal with these problems and give exact answers with proof; i.e. we can define relevant notions and give answers.
Yes! We can define what a proof is, and show in a sense that being true and having a proof are the same (Gödel's completeness theorem).
Yes! We cannot raise ourselves out of the mud: we cannot prove in our system that it does not have a contradiction (Gödel's incompleteness theorem).
Yes! We can have a general theory of mathematical theories (model theory).
Yes! We can define what it means to be computable i.e. having an algorithm (for this purpose mathematical machines were invented, and you probably have met their offspring, the computers).

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